Weak Confinement in the Ising Field Theory

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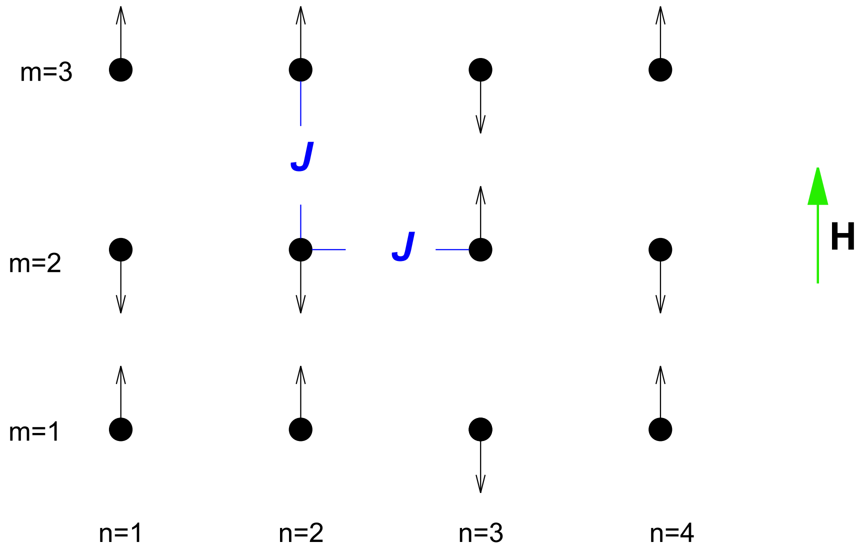
Ising model on the square lattice

$$\mathcal{E}[\sigma] = -\sum_{n=1}^{\mathcal{N}}\sum_{m=1}^{\mathcal{M}} \left(J\sigma_{m,n}\sigma_{m+1,n} + J\sigma_{m,n}\sigma_{m,n+1} + H\sigma_{m,n}\right)$$



$$\sigma_{m,n} = \pm 1$$

The lattice has $\mathcal{M}$ rows and $\mathcal{N}$ columns, $J > 0$ are the coupling constants, H is the external magnetic field.

Partition function:

$$Z_{\mathcal{M}\mathcal{N}} = \sum_{\sigma} \exp\left[-\mathcal{E}[\sigma]/(k_B T) \right],$$

Free energy:

$$f(H,T) = -k_B T \lim_{\mathcal{M},\mathcal{N}\to\infty} \frac{\ln Z_{\mathcal{M}\mathcal{N}}}{\mathcal{M}\mathcal{N}}.$$

Critical temperature (Kramers, Wannier, 1941):

$$\sinh \frac{2J}{k_B T_c} = 1.$$

Scaling, rotation invariance and universality at critical region:

$$\Delta T = (T - T_c) \to 0, \quad H \to 0.$$

Exact solution at $H = 0$, Onsager, 1944.

Ising Field Theory

Euclidean action:

$$\mathcal{A} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\psi\bar\partial\psi + \overline{\psi}\partial\overline{\psi} + im\overline{\psi}\psi\right]d^2x - h\int_{-\infty}^{\infty} d^2x\,\sigma(x),$$

$$(\mathrm{x},\mathrm{y}) = (\mathrm{x}(x),\mathrm{y}(x)), \quad z = \mathrm{x}+i\mathrm{y}, \quad \partial = \frac{\partial}{\partial z} = \frac{1}{2}(\partial_{\mathrm{x}} - i\partial_{\mathrm{y}}), \quad \bar\partial = \frac{1}{2}(\partial_{\mathrm{x}} + i\partial_{\mathrm{y}}).$$

Majorana fermions $\psi(\mathrm{x},\mathrm{y}), \overline{\psi}(\mathrm{x},\mathrm{y})$ obey at $\mathrm{y}=0$ anticommutational relations:

$$\{\overline{\psi}(\mathrm{x}),\overline{\psi}(\mathrm{x}')\} = -\{\psi(\mathrm{x}),\psi(\mathrm{x}')\} = 2\pi i\delta(\mathrm{x}-\mathrm{x}'), \quad \{\psi(\mathrm{x}),\overline{\psi}(\mathrm{x}')\} = 0.$$

Transformation under rotations $z \to e^{i\alpha}z$:

$$\psi(z) = e^{i\alpha/2}\,\psi(e^{i\alpha}z), \quad \overline{\psi}(z) = e^{-i\alpha/2}\,\overline{\psi}(e^{i}{}_{\alpha}z), \quad \sigma(z) = \sigma(e^{i\alpha}z).$$

After the Wick turn $\mathrm{y} \to it$ IFT becomes Lorentz invariant.
Parameters m and h are related with parameters ΔT and H of the original Ising model at $\Delta T \to 0$, $H \to 0$:

$$m = 2\pi\, C_\tau\, \Delta T\left(1 + O(\Delta T, H^2)\right), \quad h = C_h\, H\left(1 + O(\Delta T, H^2)\right).$$

Quantum Hamiltonian of the Ising field theory

$$\mathcal{H} = \mathcal{H}_{FF} + V.$$

Free fermionic Hamiltonian:

$$\mathcal{H}_{FF} = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\, \omega(p)\, a_p^\dagger a_p,$$

with the dispersion law $\omega(p) = \sqrt{p^2 + m^2}$, and fermionic operators a_p, $a_p^\dagger$:

$$\{a_p, a_{p'}^\dagger\} = 2\pi\delta(p - p'), \quad \{a_p, a_{p'}\} = \{a_p^\dagger, a_{p'}^\dagger\} = 0.$$

Interaction:

$$V = -h \int_{-\infty}^{\infty} d\mathrm{x}\, \sigma(\mathrm{x}).$$

Order spin operator $\sigma(\mathrm{x})$:

$$\sigma(\mathrm{x}) = \bar{\sigma} : e^{\rho(\mathrm{x})/2} : ,$$

$$\frac{\rho(\mathrm{x})}{2} = \int_{\mathrm{x}}^{\infty} d\mathrm{x}' [\chi(\mathrm{x}', \mathrm{y})\ \partial_{\mathrm{y}} \chi(\mathrm{x}', \mathrm{y})]\big|_{y=0},$$

$$\chi(\mathrm{x}, \mathrm{y}) = i \int_{-\infty}^{\infty} \frac{dp}{2\pi} \frac{e^{ipx}}{\sqrt{\omega(p)}} \left(a^{\dagger}_{-p} e^{\omega(p) y} - a_p e^{-\omega(p) y}\right),$$

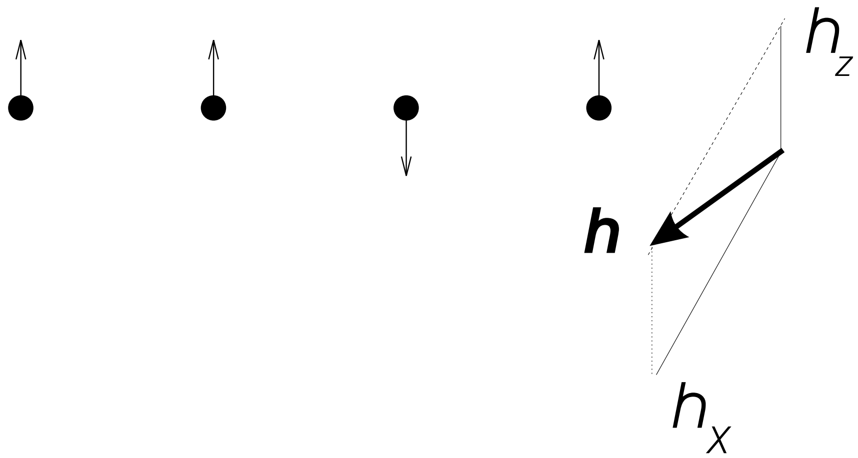
where $\bar{\sigma} = m^{1/8} 2^{1/12} e^{-1/8} A^{3/2}$ is the zero-field vacuum expectation value of the order field (spontaneous magnetization), $A = 1.28243\ldots$.

Formfactors $\langle p_1 \ldots p_N | \sigma(0) | p'_1 \ldots p'_{N'} \rangle \equiv \langle 0 | a_{p_1} \ldots a_{p_N} \sigma(0) a^{\dagger}_{p'_1} \ldots a^{\dagger}_{p'_{N'}} | 0 \rangle$ are determined by the Wick theorem with connections:

$$\frac{\langle 0 \mid \sigma(0) \mid p_1 p_2 \rangle}{\bar{\sigma}} = i \frac{\omega(p_1) - \omega(p_2)}{\sqrt{\omega(p_1)\,\omega(p_2)}} \frac{1}{p_1 + p_2},$$

$$\frac{\langle p \mid \sigma(0) \mid p' \rangle}{\bar{\sigma}} = i \frac{\omega(p) + \omega(p')}{\sqrt{\omega(p)\,\omega(p')}} \mathcal{P} \frac{1}{(p - p')}.$$

Quantum spin-$1/2$ Ising chain

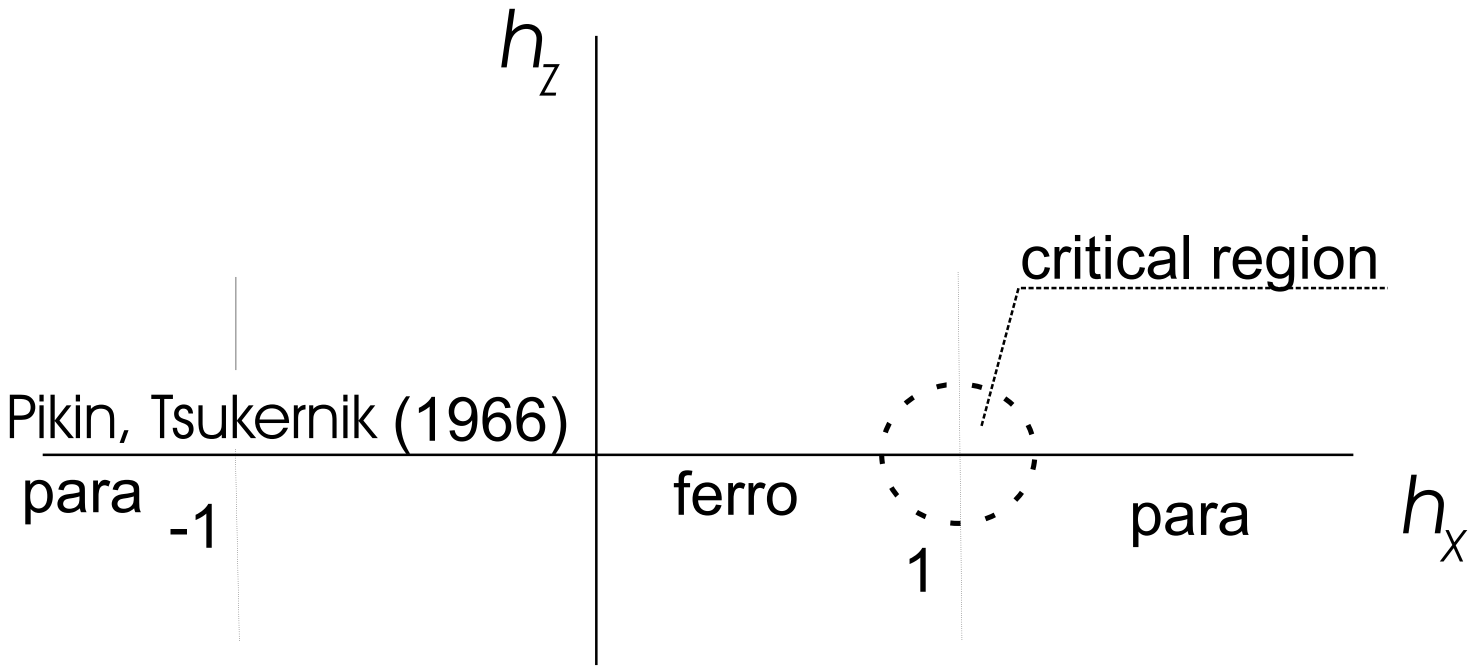


$$\mathcal{H}_{ch} = -\sum_{n=1}^{\mathcal{N}} (\sigma_n^z \, \sigma_{n+1}^z + h_x \sigma_n^x + h_z \sigma_n^z)$$

Here σ_n^x, σ_n^z are the Pauli matrices relating to the n-th site of the chain:

$$\sigma_n^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_n^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Quantum Ising spin chain phase diagram at $T = 0$

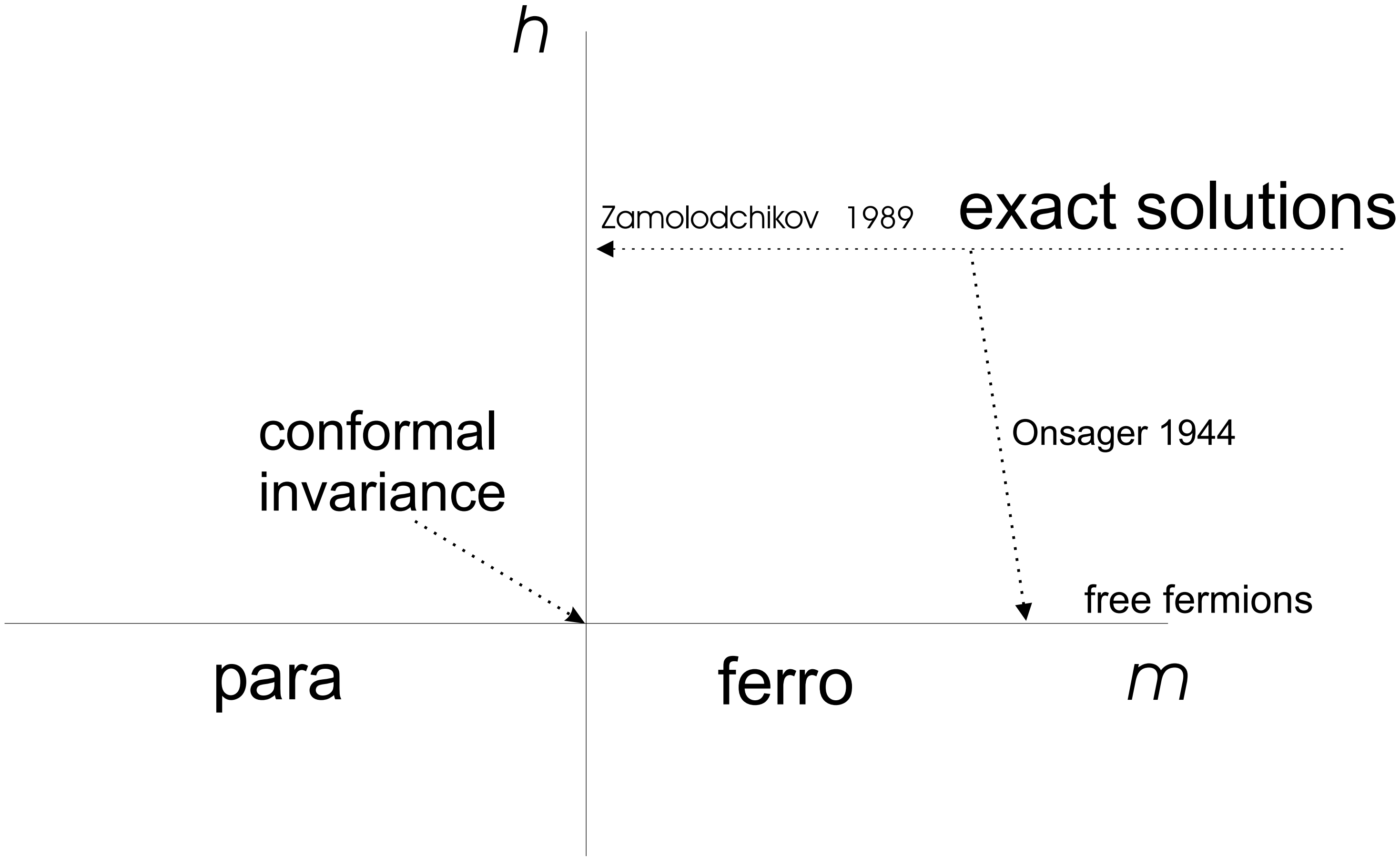


In the critical region quantum Ising spin-1/2 chain is equivalent to the Ising field theory.

Parameter correspondence:

$$m = (1 - h_x)A_m, \quad h = h_z A_h.$$

Ising field theory phase diagram



Physics of Ising field theory is determined by the single scaling parameter η:

$$\eta = \frac{m}{h^{8/15}}$$

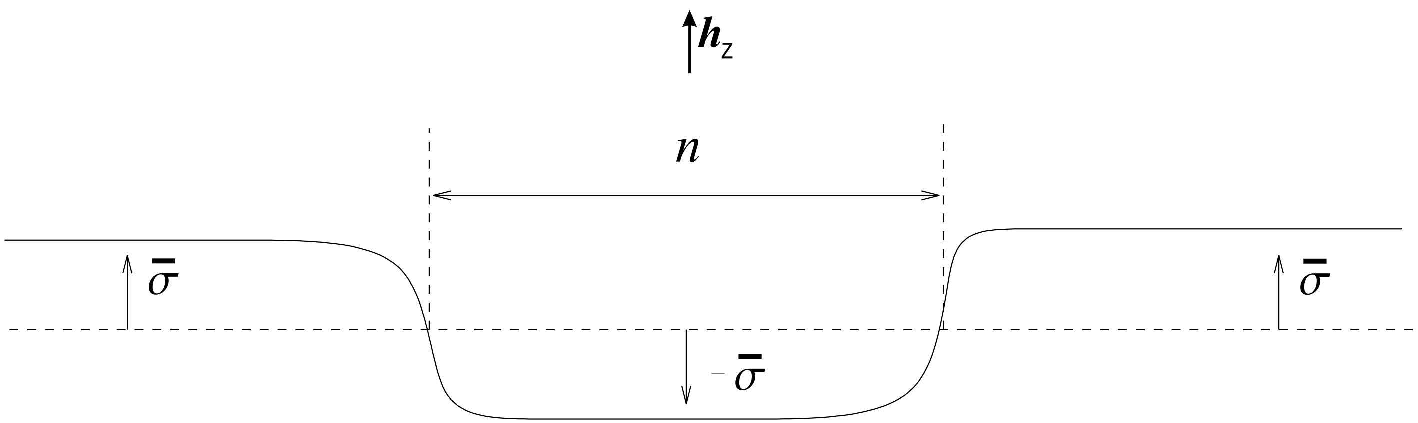
Small-h ferromagnetic regime. Confinement of fermions.

$$\mathcal{H}_{ch} = -\sum_{j=1}^{\mathcal{N}} (\sigma_j^z \sigma_{j+1}^z + h_x \sigma_j^x + h_z \sigma_j^z)$$

1. $h_z = 0$, $h_x < 1$. Two ferromagnetic ground states $|\Phi_\uparrow(0)\rangle$ and $|\Phi_\downarrow(0)\rangle$ with spontaneous magnetizations $\langle \sigma_n^z \rangle = +\bar{\sigma}$ and $\langle \sigma_n^z \rangle = -\bar{\sigma}$ have the same energy $E_\uparrow(0) = E_\downarrow(0)$. Elementary excitations (free fermions) are the domain walls, interpolating between the two degenerate vacua.

2. $1 \gg h_z > 0$, $h_x < 1$. Degeneration is removed: $|\Phi_\uparrow(h)\rangle$ is the ground state, $|\Phi_\downarrow(h)\rangle$ is the metastable state; $E_\uparrow(h_z) - E_\downarrow(h_z) \approx -2\bar{\sigma} h_z \mathcal{N}$. Two domain walls **attract** one another with the energy $2\bar{\sigma} h_z n$. An isolated domain wall gains infinite energy. Elementary excitations now are coupled pairs of fermions.

Experiment: M. Kenzelman *et al.*, Phys. Rev. B **71**, 094411 (2005).



Alternative interpretation. Ferromagnetic Ising field theory with $h > 0$ gives a $(1+1)$-dimensional relativistic model of quark confinement.

Fermions are the "quarks". Coupled pairs of fermions are the "mesons".

What energy spectrum have the "mesons" in IFT?

Motivations:

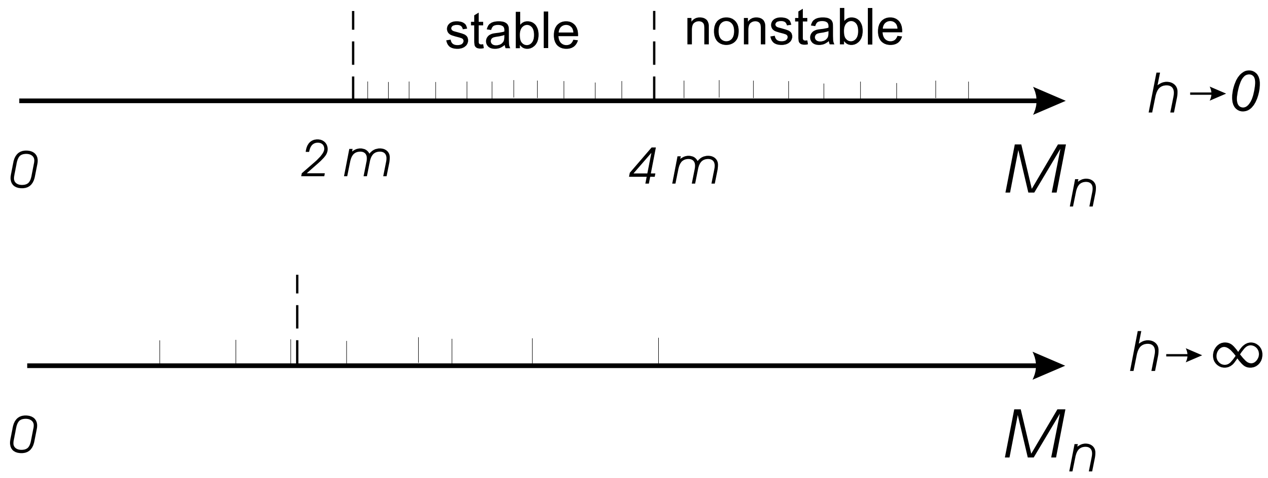
- 2D statistical mechanics: universality of the Ising fixed point, described by IFT;
- 1D condense matter: IFT describes exotic excitations in magnetic spin chains;
- high-energy physics: IFT gives a nice relativistic model of quark confinement.

The meson energy spectrum $E_n(p) = (M_n^2 + p^2)^{1/2}$ is determined by the eigenvalue problem:

$$\mathcal{H} \;|\,\Phi_n(p)\rangle = (E_n(p) + E_{\rm vac})\,|\,\Phi_n(p)\rangle, \quad P\,|\,\Phi_n(p)\rangle = p\,|\,\Phi_n(p)\rangle,$$

where P is the momentum operator, $E_{\rm vac}$ is the ground state energy, and M_n is the meson mass.

Meson mass spectrum



Meson mass spectrum at $|h| \to \infty$. (Zamolodchikov, 1989)

$$M_1 = (4,40490857...)\,|h|^{8/15}, \qquad M_2 = 2\,M_1 \cos\tfrac{\pi}{5} = (1,618...)\,M_1,$$

$$M_3 = 2\,M_1 \cos\tfrac{\pi}{30} = (1,989...)\,M_1, \qquad M_4 = 2\,M_2 \cos\tfrac{7\pi}{30} = (2,405...)\,M_1,$$

$$M_5 = 2\,M_2 \cos\tfrac{2\pi}{15} = (2,956...)\,M_1, \qquad M_6 = 2\,M_2 \cos\tfrac{\pi}{30} = (3,218...)\,M_1,$$

$$M_7 = 4\,M_2 \cos\tfrac{\pi}{5} \cos\tfrac{7\pi}{30} = (3,891...)\,M_1,$$

$$M_8 = 4\,M_2 \cos\tfrac{\pi}{5} \cos\tfrac{2\pi}{15} = (4,783...)\,M_1.$$

Singular perturbation theory for the ferromagnetic Ising field theory at small $h > 0$.

$$\mathcal{H} = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\, \omega(p)\, a_p^\dagger a_p - h \int dx\, \sigma(x),$$

with free fermionic spectrum $\omega(p) = (p^2 + m^2)^{1/2}$.

The meson masses spectrum M_n is determined by the eigenvalue problem:

$$\mathcal{H} \; | \, \Phi_n \rangle = (M_n + E_{\text{vac}}) \, | \, \Phi_n \rangle, \quad P \, | \, \Phi_n \rangle = 0.$$

Two-quark approximation:

$$\mathcal{P}_2 \mathcal{H} \mathcal{P}_2 \, | \, \Phi_n \rangle = M_n \; | \, \Phi_n \rangle,$$

where $\mathcal{P}_2$ is the orthogonal projector onto the two-quark subspace $\mathcal{F}_2$ of the Fock space $\mathcal{F}$.

Bethe-Salpeter equation. (Fonseca, Zamolodchikov, 2003)

$$[2\,\omega(p) - M_n]\,\Phi_n(p) = \bar\sigma\, h \int_{-\infty}^{\infty} \frac{1}{\omega(p)\omega(p')} \cdot$$

$$\left[\left(\frac{\omega(p)+\omega(p')}{p-p'}\right)^2 + \frac{1}{2}\frac{p\,p'}{\omega(p)\omega(p')}\right]\Phi_n(p')\,\frac{dp'}{2\pi}.$$

The principal value integral is implied. The two-quark wave function in the rest frame in the momentum representation is defined via

$$\Phi_n(p) \equiv L^{-1/2}\langle -p, p \mid \Phi_n\rangle,$$

The function $\Phi_n(p)$ is odd $\Phi_n(-p) = -\Phi_n(p)$, and should be normalized as

$$\int_0^{\infty} \frac{dp}{2\pi} \mid \Phi_n(p) \mid^2 = 1, \quad \text{if } \langle \Phi_n \mid \Phi_n\rangle = 1.$$

Bethe-Salpeter equation in configuration space.

$$(2\hat{K} - M_n)\phi_n(\mathrm{x}) = -2\bar{\sigma}h|\mathrm{x}|\,\phi_n(\mathrm{x}) + 2\bar{\sigma}h\hat{U}\,\phi_n(\mathrm{x}).$$

Here $\phi_n(\mathrm{x})$ denotes the configuration-space wave function

$$\phi_n(\mathrm{x}) = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\, e^{ip\mathrm{x}} \Phi_n(p),$$

$$\phi_n(-\mathrm{x}) = -\phi_n(\mathrm{x}),$$

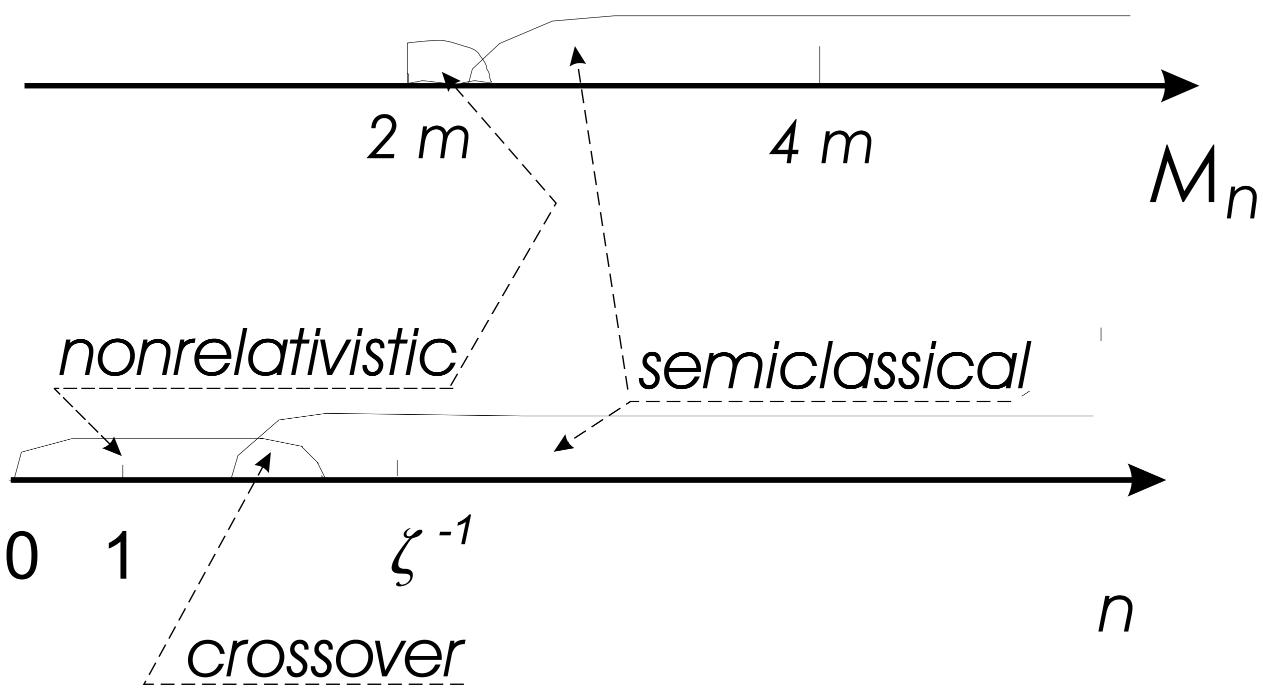
and the integral operators $\hat{K}$ and $\hat{U}$ have the kernels

$$K(\mathrm{x}, \mathrm{x}') = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\, e^{ip(\mathrm{x}-\mathrm{x}')} \sqrt{p^2 + m^2},$$

$$U(\mathrm{x}, \mathrm{x}') = \frac{1}{2} \iint_{-\infty}^{\infty} \frac{dp\,dp'}{4\pi^2}\, \frac{\exp[i(p\mathrm{x} - p'\mathrm{x}')]}{\omega(p)\omega(p')} \cdot$$

$$\left[\left(\frac{\omega(p) - \omega(p')}{p - p'} \right)^2 + \frac{1}{2} \frac{p\,p'}{\omega(p)\omega(p')} \right].$$

Nonrelativistic, semiclassical and crossover regions



Here n is the serial number of the meson state: $M_n < M_{n+1}, \quad n = 1, 2, 3, ...$

- Nonrelativistic: $n \ll \zeta^{-1}$, where $\zeta = 2\bar{\sigma}h/m^2$.
  Fonseca, Zamolodchikov (2003).
- Semiclassical: $n \gg 1$. R (2005).
- Crossover: $1 \ll n \ll \zeta^{-1}$.

Nonrelativistic approximation $n \ll \zeta^{-1}$

Relativistic kinetic energy operator $\hat{K}$ is replaced by its nonrelativistic counterpart

$$\hat{K} \to m - \frac{1}{2m}\frac{d^2}{d\mathrm{x}^2} - \frac{1}{8m^3}\frac{d^4}{d\mathrm{x}^4} - \frac{1}{16m^5}\frac{d^6}{d\mathrm{x}^6} + \cdots,$$

In this approximation the Bethe-Salpeter equation reduces to the perturbed Airy equation

$$\left(2m - M_n - \frac{1}{m}\frac{d^2}{d\mathrm{x}^2}\right)\phi_n(\mathrm{x}) + 2\bar{\sigma}h|\mathrm{x}|\,\phi_n(\mathrm{x}) + \delta\hat{V}\phi_n(\mathrm{x}) = 0, \quad \phi_n(-\mathrm{x}) = \phi_n(\mathrm{x}),$$

yielding (Fonseca, Zamolodchikov 2003):

$$\frac{M_n - 2m}{m} = \zeta^{2/3} z_n - \frac{\zeta^{4/3}}{20} z_n^2 + \zeta^2\left(\frac{11\, z_n^3}{1400} - \frac{57}{280}\right) + O(\zeta^{8/3}),$$

where ζ is a dimensionless parameter proportional to h: $\zeta = 2\bar{\sigma}h/m^2$, and $-z_n, n = 1, 2, \ldots$ are zeros of the Airy function, $\mathrm{Ai}(-z_n) = 0$.

The leading term reproduces the old result of McCoy and Wu (1974).

Semiclassical (WKB) mass spectrum $n \gg 1$:

$$M_n = m\, u_n + \zeta^2 A_n^{(2)} + \zeta^3 A_n^{(3)} + ...,$$

where u_n solves equation

$$\frac{u_n}{2}\left(\frac{u_n^2}{4} - 1\right)^{1/2} - \operatorname{arccosh}(u_n/2) = \left(n - \frac{1}{4}\right)\pi\,\zeta,$$

and

$$A_n^{(2)} = \frac{m}{u_n^3}\left[\frac{40}{3(u_n^2-4)^2} + \frac{6}{u_n^2-4} - 1\right].$$

In the crossover region $1 \ll n \ll \zeta^{-1}$, the both nonrelativistic and the semiclassical expansions can be reduced to the same form:

$$\frac{M_n - 2m}{m} = \zeta^{2/3}\left[b_n + \frac{5}{48}b_n^{-2} + O(b_n^{-5})\right] + \zeta^{4/3}\left[-\frac{b_n^2}{20} - \frac{b_n^{-1}}{96} + O(b_n^{-4})\right] + \zeta^{6/3}\left[\frac{11\, b_n^3}{1400} - \frac{901}{4480} + O(b_n^{-3})\right] + O(\zeta^{8/3}),$$

where $b_n = \left[\frac{3\pi}{8}(4n-1)\right]^{2/3} \gg 1$.

$$(2\hat{K} - M_n)\phi_n(\mathrm{x}) = -2\bar{\sigma}h|\mathrm{x}|\,\phi_n(\mathrm{x}) + 2\bar{\sigma}h\hat{U}\,\phi_n(\mathrm{x}).$$

where

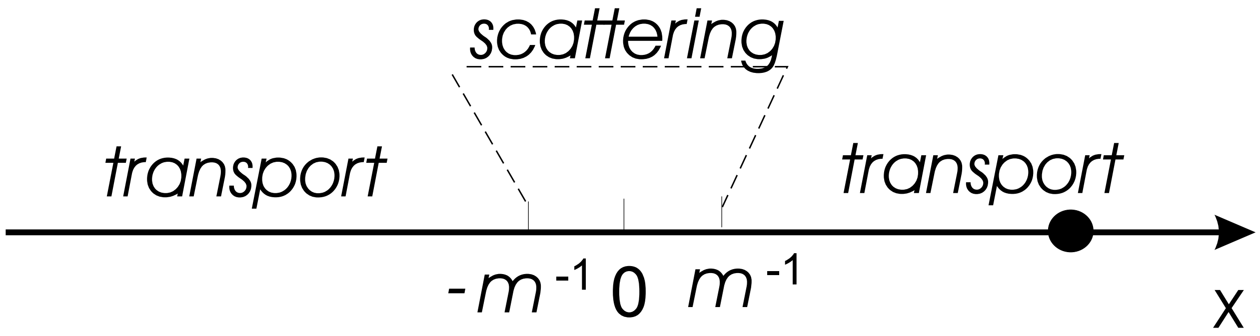
$$\hat{K}\phi_n(x) = \int_{-\infty}^{\infty} dx'\,K(x-x')\,\phi_n(x'),$$

$$\hat{U}\phi_n(x) = \int_{-\infty}^{\infty} dx'\,U(x,x')\,\phi_n(x'),$$

$$K(x-x') = \int_{-\infty}^{\infty} \frac{dp}{2\pi}\,e^{ip(x-x')}\sqrt{p^2+m^2},$$

$$\phi_n(x) = -\phi_n(-x), \quad h \to 0, \quad n \gg 1,$$

and $U(x,x')$ is localized in the region $\max(|x|,|x'|) \lesssim m^{-1}$.



$$m^{-1} \ll a \ll (m\zeta)^{-1}$$

In the transport regions quarks move like classical particles, in the inner region quantum scattering occurs.
Analogy with magnetic breakdown in normal metals.
1. Right transport region $\mathrm{x} > a$: $\phi_n(\mathrm{x}) \cong C\,\phi_r(\mathrm{x}, \mathrm{M_n})$, where $\phi_r(\mathrm{x}, \mathrm{M_n})$ solves equation

$$(2\hat{K} - M_n)\phi_r(\mathrm{x}, M_n) = -2\,\bar{\sigma}\,h\,\mathrm{x}\,\phi_r(\mathrm{x}, M_n)$$

in $\mathbb{R}$.
In momentum representation this equation reads as

$$(2\sqrt{p^2 + m^2} - M_n)\Phi_r(p, M_n) = 2i\,h\,\bar{\sigma}\,\partial_p \Phi_r(p, M_n),$$

providing

$$\phi_r(\mathrm{x}, M_n) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} \exp\left[\frac{i\big(f(p) - M_n\,p\big)}{2h\bar{\sigma}} + ip\mathrm{x}\right],$$

$$\text{with } f(p) = 2\int_0^p dp'\,\omega(p').$$

2. Left transport region $\mathrm{x} < -a$: $\phi_n(\mathrm{x}) \cong -C\,\phi_r(-\mathrm{x}, \mathrm{M_n})$.
3. Scattering region

$$\phi_n(\mathrm{x}) = \phi_n^{(0)}(\mathrm{x}) + \phi_n^{(1)}(\mathrm{x}) + \ldots$$

$$\phi_n^{(0)}(\mathrm{x}) = A \sin p_0 \mathrm{x}, \quad \phi_n^{(1)}(\mathrm{x}) \sim h,$$

where $p_0 = [(M_n/2)^2 - m^2]^{1/2}$. Joining at $\mathrm{x} \simeq \pm a$ solutions of different regions provides us the condition, which determines the discrete semiclassical mass spectrum M_n:

$$M_n = m\, u_n + M_n^{(2)} + O(\zeta^3),$$

where u_n solves equation

$$\frac{u_n}{2}\left(\frac{u_n^2}{4} - 1\right)^{1/2} - \operatorname{arccosh}(u_n/2) = \left(n - \frac{1}{4}\right)\pi\,\zeta,$$

and

$$M_n^{(2)} = \frac{\zeta^2\, m}{u_n^3}\left[\frac{40}{3(u_n^2-4)^2} + \frac{6}{u_n^2-4} - 1\right].$$

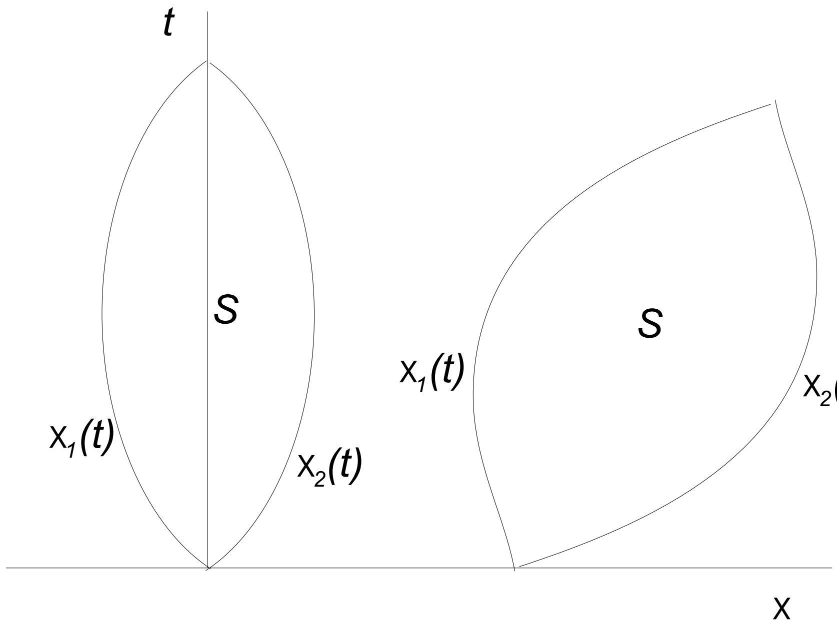
Bohr-Sommerfeld quantization rule

Consider two interacting particles with coordinates $\mathrm{x}_1(t)$, $\mathrm{x}_2(t) \in \mathbb{R}$ described by the classical Lorentz invariant action

$$\mathcal{A}_{\mathrm{cl}} = -m \int_0^{t_m} dt[(1-\dot{\mathrm{x}}_1^2)^{1/2} + (1-\dot{\mathrm{x}}_2^2)^{1/2}] - 2h\bar{\sigma}S,$$

$$\text{with } S = \int_0^{t_m} dt\,[\mathrm{x}_2(t) - \mathrm{x}_1(t)],$$

under the following constrains : $\mathrm{x}_1(0) = \mathrm{x}_2(0)$, $\mathrm{x}_1(t_m) = \mathrm{x}_2(t_m)$, and $\mathrm{x}_1(t) < \mathrm{x}_2(t)$ for $0 < t < t_m$. Typical world paths of the particles look like:



The Bohr-Sommerfeld quantization condition can be written in a relativistic invariant form:

$$2h\,\bar{\sigma}\,S = 2\pi\Big(n - \frac{1}{4}\Big).$$

It leads to the semiclassical mass spectrum $M_n = m\,u_n$ of the pair with the previous meaning of u_n:

$$\frac{u_n}{2}\left(\frac{u_n^2}{4} - 1\right)^{1/2} - \operatorname{arccosh}(u_n/2) = \left(n - \frac{1}{4}\right)\pi\,\zeta.$$

Multi-quark corrections

Exact eigenvalue problem:

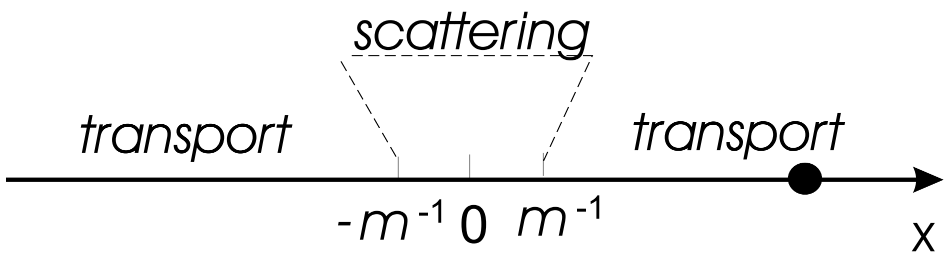
$$\mathcal{H}\;\,|\,\Phi_n\rangle = (M_n + E_{\text{vac}})\,|\,\Phi_n\rangle, \quad P\,|\,\Phi_n\rangle = 0,$$

The exact meson eigenvector $|\Phi_n\rangle$ contains four-quark, six-quark, ... contributions, which are ignored in the two-quark approximation.

Taking into account, multi-quark corrections modify the semiclassical meson spectrum $M_n = m\,u_n + \zeta^2 A_n^{(2)} + \zeta^3 A_n^{(3)} + \ldots$, starting from the second order in ζ.

Two-quark approximation:

$$\mathcal{P}_2 \mathcal{H} \mathcal{P}_2 \mid \Phi_n\rangle = M_n \mid \Phi_n\rangle, \quad P \mid \Phi_n\rangle = 0, \quad |\Phi_n\rangle \in \mathcal{F}_2,$$



- Transport regions: renormalization of $\omega(p)$ and $h\bar{\sigma}$. Renormalization of the quark mass (Fonseca, Zamolodchikov 2003): $\delta m \approx 0,03550540475\,\zeta^2\, m$.
- Scattering region: Collision remains elastic.
- $M_n > 2M_1$. Nonelastic channels open. The meson decay rate can be estimated from the Fermi's golden rule:

$$\Gamma_n = 2\pi \sum_{\text{out}} \mid \langle \text{out} \mid V \mid \Phi_n\rangle \mid^2 \delta(m\, u_n - E_{\text{out}}).$$

where $\mid \Phi_n\rangle$ is the meson eigenvector in the two-quark approximation, and $\langle \text{out} \mid = \langle p_{2j} \dots p_1 \mid$.

$$\Gamma_n = \frac{h^3 \bar{\sigma}\, u_n}{m(u_n^2 - 4)} \sum_{j=2}^{j_m} \frac{1}{(2j)!} \int_{-\infty}^{\infty} \frac{dp_1 ... dp_{2j}}{(2\pi)^{2j-2}}\, \delta(p_1 + \cdots + p_{2j})$$

$$\delta\left[\omega(p_1) + \cdots + \omega(p_{2j}) - m\, u_n\right] \mid \langle p_{2j} ... p_1 \mid \sigma(0) \mid p_0, -p_0 \rangle \mid^2 + O(\zeta^4),$$

where:

$u_n = M_n/m$,

$\pm p_0 = \pm\sqrt{(M_n/2)^2 - m^2}$ are the momenta of two colliding quarks,

$j_m = [u_n/2]$ stays for the integer part of $u_n/2$.

The extra h-factor here reflects, that quarks spend almost all the time in the transport regions, and only rarely come up in the scattering region, from which the decay then occurs.

Conclusions

Semiclassical excitation spectrum in the Ising field theory is obtained in the weak confinement regime perturbatively in applied field h.

- It is shown, that the two-fermionic approximation is sufficient to determine the *entire spectrum* up to the first order in h. The many body effects are important in the second order.
- For energies above the stability threshold, the excitation decay width has been determined in the leading h^3order.