Experimental and numerical studies of thermal convection have shown that sufficiently vigorous convective flows exhibit a large-scale thermal wind component sweeping along small-scale thermal boundary layer instabilities. A characteristic feature of these flows is an intermittent behavior in the form of irregular reversals in the orientation of the large-scale circulation. There have been several attempts toward a better understanding and description of the phenomenon of flow reversals, but so far most of these models are based on a statistical analysis of few-point measurements or on simplified theoretical assumptions. The analysis of long-term data sets \((>5 \times 10^5\) turnover times \(\tau_t = d/u_{rms}\)) obtained by numerical simulations of turbulent two-dimensional Rayleigh-Bénard convection allows us to get a more comprehensive view of the spatio-temporal flow behavior. By means of a global statistical analysis of the characteristic spatial modes of the flow we extract information about the stability of dominant large-scale modes as well as the reversal paths in state subspace. We examine probability density functions and drift vector fields of two-dimensional state subspaces spanned by different large-scale spatial modes. This also provides information about the coexistence of dominant modes.

I. INTRODUCTION

Even though Rayleigh-Bénard convection, i.e., a buoyancy-driven flow within a fluid layer heated from below and cooled from above, is a classical and intensively studied hydrodynamical problem, it still comprises several open questions (see, e.g., a recent review paper by Ahlers et al. [1]).

Experimental (e.g., Refs. [2–4]) and numerical studies (e.g., Refs. [5,6]) of thermal convection have shown that sufficiently vigorous convective flows exhibit a large-scale thermal wind component sweeping along small-scale thermal boundary layer instabilities. A characteristic but still poorly understood feature of these flows is an intermittent behavior in the form of irregular reversals in the orientation of the large-scale circulation (LSC) observed in several experimental [7–9] and numerical studies [10–12]. In three dimensions one has to distinguish two reversal scenarios: The first is characterized by cessation of a partly stable LSC followed by an effective reorganization of the system into a large-scale flow with reversed orientation. The second mechanism results from an azimuthal drift by 180 degrees of the LSC. In this paper we will discuss two-dimensional (2D) convection, so that the only mechanism for a reversal is cessation of the flow structure, and the azimuthal drift will not be addressed further. Such reversals in convective systems can also be observed in nature. Prominent examples are the reversing geodynamo and reversals in the wind direction of the atmosphere. There have been several attempts toward a better understanding and description of the phenomenon of flow reversals, but so far most of these models are based on statistical analysis of few-point measurements or on simplified theoretical models [13–16]. These models are introduced phenomenologically on the basis of chaotic [13] or stochastic [14–16] ordinary differential equations with two metastable states and are not based directly on an analysis of the Rayleigh-Bénard flow fields.

In contrast to laboratory studies, direct numerical simulations (DNSs) provide a comprehensive picture of the spatial field structure and are therefore very suitable for examining structural reorganizations within convective flows. The DNS, however, is limited in its spatio-temporal resolution due to computational restrictions.

We were originally motivated by studying convective flows within planetary mantles. Due to the high viscosity of mantle rocks, the value of the Prandtl number given by \(\text{Pr} = \nu/\kappa\) (\(\nu, \kappa\) denote kinematic viscosity and thermal diffusivity, respectively) is estimated to be around \(10^2\) [17]. In this case it is reasonable to consider the infinite Prandtl number limit, implying that mechanical inertia is neglected in the momentum equation. Intrinsically, infinite Prandtl number convection does not contain toroidal motion (horizontal vortices). Schmalzl et al. [18] have shown that in this case convection can be reasonably well approximated by a 2D model. This vastly reduces the computational effort of treating such a system numerically and enables us to examine long-term data sets \((>5 \times 10^5\) turnover times \(\tau_t = d/u_{rms}\)), being indispensable for a quantitative statistical analysis. Even though mechanical inertia is neglected, i.e., the Reynolds number of the system is zero, convection at a sufficiently high Rayleigh number shows turbulent behavior including intermittent behavior in the form of reversals in the LSC [11]. Most studies examine turbulent Rayleigh-Bénard convection applying rigid boundary conditions, and therefore several phenomena are observed related to viscous boundary layers [19,20]. For example, reversals of LSC are often characterized by the appearance of corner flows, i.e., counterrotating flow structures generated by the viscous boundary layer [21]. Though stress-free boundary conditions are experimentally difficult to realize, they are a good approach for many systems in nature. Many examples can be found in geophysics or astrophysics, such as convection in stars, in the ocean, or in the earth’s mantle. In our simulations we investigate
Rayleigh-Bénard convection under stress-free boundary conditions. In such a configuration, viscous boundary layers do not exist, i.e., their thickness is zero. Assuming the Boussinesq approximation [22], the governing set of nondimensional model equations can be written as

\[ \begin{align*}
0 &= -\nabla \bar{p} + \Delta \mathbf{u} + RaTe_z, \\
\mathbf{v} \cdot \mathbf{u} &= 0, \\
\partial_t T + \mathbf{u} \cdot \nabla T &= \Delta T,
\end{align*} \tag{1-3} \]

where \( \mathbf{u} \) is the velocity vector, \( \bar{p} \) the pressure due to the hydrostatic component, \( T \) the temperature, and \( e_z \) the unit vector in the \( z \) direction. The set of equations constitutes the conservation of momentum, mass, and energy. The equations have been nondimensionalized by means of a characteristic length scale \( d \) (the system height), the temperature difference \( \Delta T = T_{\text{top}} - T_{\text{bot}} \), and the thermal diffusion time \( \tau_\delta = d^2/\kappa \). In this case the only appearing control parameter is the Rayleigh number \( Ra = a g \Delta T d^3/\nu \kappa \), specifying the relative strength of buoyancy-driven convection relative to diffusion. Here \( \alpha \) denotes the thermal expansion, and \( g \) the acceleration of gravity.

Our numerical study aims at a global statistical analysis of high Rayleigh number convection with respect to the stability of dominant spatial modes of LSC as well as characteristic reversal paths.

This paper is organized as follows: First, we will give a short description of the applied numerical method and the considered model parameters in Sec. II. Section III outlines the characteristics of the present LSC and introduces the underlying method of primary mode decomposition applied for the separation of LSC from superimposed plume dynamics. In Sec. IV we use this method to analyze a characteristic path of a reorientation in the LSC by means of a selected reversal sequence. A more global statistical analysis is then given in Sec. V, where we examine probability density functions (PDFs) and drift vector fields of 2D state subspaces spanned by different large-scale modes. This provides information about the coexistence as well as transition paths of dominant large-scale modes. In Sec. VI we address the importance of the large-scale modes and the plume dynamics for the global heat transport followed by a summary of the results (Sec. VII).

II. NUMERICAL MODEL

We use a 2D convection model by Trompert and Hansen [23], based on a finite-volume discretization of the governing equations (1)–(3), at a value of \( Ra = 10^8 \) with an aspect ratio of \( \Gamma = 2 \). Stress-free boundary conditions have been applied on all boundaries. The nondimensional temperatures at the top and bottom are kept constant at zero and one, respectively, whereas the side walls are assumed to be insulating. As initial condition we have chosen the conductive state, with a superimposed high-frequency random perturbation to inhibit an induced particular evolution. The grid resolution is \( N_x = 256 \) grid points in the horizontal direction and \( N_z = 128 \) in the vertical direction. Thermal convection at high Rayleigh numbers is characterized by the appearance of thin thermal boundary layers at the top and bottom walls, which have to be appropriately resolved. Therefore, we use a grid refinement in the vertical direction toward the top and bottom walls defined by roots of Chebyshev polynomials [24]. In our case, at least 10 grid points are then lying within the thermal boundary layer region.

III. LARGE-SCALE CIRCULATION AND PRIMARY MODE DECOMPOSITION

The convective flow of our long-term simulation at \( Ra = 10^8 \) and aspect ratio \( \Gamma = 2 \) features a developed LSC superimposed by small-scale boundary layer instabilities in the form of thin plume-like up and down wellings. These boundary layer instabilities are irregular, disconnected structures carried by the thermal wind. The LSC turns out to be only temporarily stable by switching predominantly between clockwise or anti-clockwise orientated one- and two-cell patterns, respectively. Breuer and Hansen [11] showed for similar model parameters that there exist four metastable states that turned out to be stationary solutions of the governing equations (1)–(3). These fixed-point solutions represent the primary large-scale flow pattern as described above. The temporal flow structure can now be interpreted by different primary modes superimposed by plumes that act in a sense as noise. To analyze the large-scale dynamics, i.e., flow reversals and primary mode selection, it is reasonable to separate the large-scale wind pattern from the small-scale plume dynamics. In more complex geometry (as e.g., Ref [25]; three-dimensional cylindrical coordinates) methods such as proper orthogonal decomposition [26] can be used to determine an orthonormal system. However, for our current study of the 2D system, an expansion in eigenfunctions of the Laplacian \( \Delta \mathbf{u} \) in the momentum equation (1) is a convenient way to characterize the stationary solutions of the velocity field qualitatively. This can be expressed by

\[ \mathbf{u}(x,z,t) = \sum_{m} \sum_{l} \xi_{m,l}(t) \hat{\mathbf{u}}_{m,l}(x,z) \]

with 2D orthonormal eigenfunctions \( \hat{\mathbf{u}}_{m,l} \) and corresponding time-dependent amplitudes \( \xi_{m,l} \). The eigenfunctions can be separated into a vertical and a horizontal component given by

\[ \begin{align*}
\hat{\mathbf{u}}_{m,l}(x,z) \cdot e_x &= \sqrt{\frac{\pi}{z_{\text{max}}}} \cos \left( \frac{l \pi z}{z_{\text{max}}} \right), \\
\hat{\mathbf{u}}_{m,l}(x,z) \cdot e_z &= \sqrt{\frac{\pi}{x_{\text{max}}}} \sin \left( \frac{m \pi x}{x_{\text{max}}} \right),
\end{align*} \]

where \( x_{\text{max}} \) and \( z_{\text{max}} \) are the length and height of the simulation domain. The eigenfunctions \( \hat{\mathbf{u}}_{m,l} \) satisfy the applied stress-free boundary conditions. Based on the two-dimensionality and the incompressibility equation (2), the vertical and horizontal component of the velocity are directly coupled as both velocity components can be derived from a stream function. Thus it suffices to examine one component. Due to orthonormality of the eigenfunctions, projection of the vertical velocity component \( u_z \) gives the amplitude corresponding to the mode \( m,l \), defined by

\[ \xi_{m,l}(t) = \int_0^{x_{\text{max}}} \int_0^{z_{\text{max}}} u_z(x,z,t) \hat{\mathbf{u}}_{m,l}(x,z) \cdot e_z \, dx \, dz. \]

As described above, the applied functions for primary mode projection are eigenfunctions of the Laplacian \( \Delta \mathbf{u} \) but not of...
the full nonlinear system given by Eqs. (1)–(3). Therefore they are independent of the Rayleigh number and do not resolve boundary layers and plume dynamics. However, the primary modes shown in Fig. 2 represent to first order the large-scale dynamics (Fig. 1). For example, the large-scale dynamics of the flow showing a one-cell LSC state [Fig. 1(a)] can approximately be represented by the $\xi_{11}$ mode [Fig. 2(a)]. An appropriate superposition of odd modes, where the velocity cancels out in the bulk but adds up in the bound, leads to a LSC state with thinner velocity boundary layers. Hence this approach will be used to separate the LSC from the plume dynamics.

IV. REVERSAL SEQUENCE AND REPRODUCTION OF THE VERTICAL VELOCITY

As stated above, a characteristic feature of high Rayleigh number convection appears to be a frequent reorientation of the LSC. This is illustrated in Fig. 3 showing the temporal evolution of the vertical velocity at two distinct measurement points, located at middepth at the left and the right side walls, respectively. One can clearly see the temporal correlation of these two measurements reflecting the large-scale dynamics of the system. A change in sign in both measurements at the side walls indicates a global reorientation of the LSC, also termed flow reversal. Concerning this matter, the prominent question is: What triggers such a reversal, and what are the characteristics of a reversal path?

In the following we will discuss exemplarily the flow evolution path during one arbitrarily selected reversal. To this end different stages of the temporal evolution of one representative reversal are displayed in Fig. 4. Initially a stable one-cell structure has formed, which leads to a high amplitude $\xi_{11}$ [Fig. 4(a)] acting as an indicator for a superposition of odd modes. This pattern is superimposed by small-scale plumes that contribute to eigenfunctions of higher orders. Due to the strong boundary layer dynamics, a few strong plumes develop from time to time. If such a plume rises fast enough to disturb the flow structure, the current flow pattern becomes unstable. After a short period of plume-dominated behavior, a two-cell pattern evolves with respect to the global wind [Fig. 4(b)]. In terms of the amplitudes this means that there is a strong $\xi_{21}$ and vanishing $\xi_{11}$. This relatively stable flow pattern with two equally sized cells changes its structure after a few circulation periods by varying the size of both convection cells. This time-dependent structure becomes unstable evolving toward a three-cell pattern with one big cell in the middle and two small cells at each side of the main cell. With increasing time, the flow librates between a one- and two-cell pattern of different cell size. The amplitudes $\xi_{11}$ and $\xi_{21}$ are reflecting this behavior in strong alternating fluctuations, where it is possible that both amplitudes are nonzero [Fig. 4(c)]. Later the flow has established again a stable one-cell state but in a reversed direction and with zero $\xi_{21}$ [Fig. 4(d)]. In our simulations such a generic reversal path is the most frequent evolution of change in flow orientation. However, it is not necessary that a breakdown of a stable one-cell state always leads to a reversal of flow direction. It is just as likely for the flow to establish an LSC in the same direction as before.

The temporal evolution of the vertical velocity at the side walls at middepth can be expanded in a series of eigenmodes as discussed in the theoretical section. For an adequate analysis of primary mode decomposition it is interesting to see how many modes are sufficient to describe the dynamics of the system. Figure 5 displays the comparison between the
FIG. 4. (Color online) (Left panels) Snapshots of the flow structure at distinct points in time. Shown are snapshots of the vertical velocity component $u_z$ (left), the horizontal velocity component $u_x$ (middle), and the temperature field (right). (Right panel) Temporal evolution of the vertical velocity (green (upper) line) at the left side wall at middepth and the corresponding evolution of the modes $\xi_{11}$ (solid red) and $\xi_{21}$ (dashed blue). The temporal evolution indicates a reversal of sign of a single-cell pattern. The arrows with letters a–d mark the distinct states seen in the left panel.

The temporal evolution of the vertical velocity at the side walls at middepth and a reduced model of a finite set of modes:

$$u(x,z,t) = \sum_{m=1}^{N} \xi_{m1}(t) \hat{u}_{m1}(x,z)$$

for $N \leq 5$.

For a decreasing number of participating modes the amplitude of the vertical velocity component is decreased, but the correlation remains robust for all $N > 1$. Hence the global dynamics of the system as well as the reversal behavior can be represented by only the first two modes, $\xi_{11}$ and $\xi_{21}$. This means that the first two modes act as the leading modes in a superposition. However, if more modes are incorporated, there is a better agreement of the amplitudes.

FIG. 5. (Color online) Temporal evolution of the vertical velocity component $u_z$ (dashed blue) and of a reduced model (solid red), which contains only a few basic modes. (a) $\xi_{11}$ to $\xi_{21}$, (b) $\xi_{11}$ and $\xi_{21}$, and (c) $\xi_{11}$. For decreasing $N$ the amplitude of the reproduced vertical velocity component is decreasing, but even the $N = 2$ case strongly correlates with the full data set.

V. PDFS AND DRIFT VECTOR FIELDS OF 2D STATE SUBSPACES

In the previous section we discussed exemplarily one reversal path. We showed that in this specific case the change from a one-cell LSC to a one-cell circulation with reversed orientation is not a direct transition but follows a path over a dominant two-cell state. To answer the question if this is a characteristic behavior, it is insightful to examine the 2D PDF of the amplitudes of the dominant modes $\xi_{11}$ and $\xi_{21}$. This PDF of the 2D subspace allows us to extract the statistical dependency with respect to these two primary large-scale modes. However, due to the restriction of dimension in state subspace we cannot extract information of the other modes at the same time. This means that this projection allows us only to discuss the dependency of two modes on each other. To study the dynamics of transitions between the two states, we estimate finite-time drift vector fields of different pairs of amplitudes. Similar as in the theory of stochastic processes [27], these may be defined as

$$D(\tilde{q}) = \frac{1}{\tau} (q(t + \tau) - q(t) | q(t) = \tilde{q}),$$

where $q(t) = [\xi_{ml}(t), \xi_{nk}(t)]$ is the position in the state subspace and $\tau$ is taken much smaller than the average difference between two reversals $\tau_r$. We have checked that the topological structure of the vector field is robust as long as the condition $\tau \ll \tau_r$ holds. The interpretation of this quantity is particularly intuitive; given a certain pair of values $(\xi_{ml}, \xi_{nk})$ in the state subspace, the drift vector field indicates the direction of the temporal evolution of these two amplitudes on the...
The PDF extracted from the primary modes $\xi_{11}$ and $\xi_{21}$ in Fig. 6 corresponding to the one- and two-cell flow pattern shows four isolated maxima indicating the most probable configurations, two distinct maxima with respect to the one-cell circulation with zero $\xi_{21}$, and two distinct maxima of the two-cell circulation with zero $\xi_{11}$. This indicates that both flow structures do not occur simultaneously. Additionally, the orientation of the drift vector field marked with the white arrows reveals the statistical motion in the state subspace. There are four stable fixed points that correspond to the maxima of the PDF. Due to an unstable fixed point the system is pushed away from the origin; this implies that a reversal in the orientation of one of these modes always goes along with the appearance of the second dominant mode and vice versa. The absolute value of the vector field implies how strong the system is attracted into the fixed-point solution. Due to this absolute value in combination with the higher probability of $\xi_{11}$ it is clear that the one-cell circulation is the most dominant flow pattern. Of course, it is also interesting to study relations involving higher-order modes. Different state subspaces of higher-order modes are shown in Fig. 7. Contrary to the $\xi_{11}, \xi_{21}$ subspace the amplitudes $\xi_{11}$ and $\xi_{31}$ are not independent of each other, as Fig. 7(a) reveals. Both amplitudes in this state subspace are nonzero at the most probable configuration, which indicates that the one- and three-cell pattern exist simultaneously. This also indicates that a one-cell LSC structure not only consists of one mode but of the superposition of odd modes. However, $\xi_{11}$ is the strongest amplitude. The drift vector field indicates two stable attractors that conform with the most probable configurations, which are not subdivided into distinct maxima. In the origin the drift vector field specifies a saddle point that corresponds to the appearance of other modes. Similar to the one- and three-cell

VI. PLUME DYNAMICS AND HEAT TRANSPORT

Due to the analysis of the state subspace we are able to describe the flow phenomena in terms of amplitudes extracted from spatial modes. In order to extract more information about the flow structures in the $\xi_{11}, \xi_{21}$ subspace (Fig. 6), typical flow fields belonging to different points in the $\xi_{11}, \xi_{21}$ plane are displayed in Fig. 8. This establishes a connection between different points in state subspace and the corresponding flow configurations and allows us to discuss the statistical results with respect to dynamical features of the flow. At the distinct maxima, indicating the most probable configuration with respect to the one-cell pattern, a flow circulation with only a few plume-like structures has developed [Fig. 8(a)]. At this stage the plumes do not reach far into the bulk. The thermal instabilities travel along the boundary layer and do not grow until they cross approximately two-thirds of the simulation domain. With increasing $\xi_{11}$ the flow gets more nonstationary. The plumes separate earlier from the boundary layer and interact with the bulk more frequently [Fig. 8(b)]. Both velocity components in Figs. 8(a) and 8(b) indicate that one circulation cell covers the whole simulation domain. During the transition between a one- and two-cell structure both amplitudes are nonvanishing, and hence the flow is a superposition of a
one- and a two-cell pattern. Such a flow pattern with one big and one small convection cell is displayed in Fig. 8(c). Due to a high magnitude of the drift vector field there is a strong variation in the flow structure. Similar to Fig. 8(b) these variations are related to an increased plume activity. This points out that the high magnitude of the drift vector field is linked to a strong interaction of the plumes with the flow. Hence, a transition between two fixed points always goes along with the development of a strong plume, which perturbs the stable flow pattern. In the case where both dominant modes $\xi_{11}$ and $\xi_{21}$ are equal to zero, the flow has to organize in higher-order modes. The plume-dominated dynamics is strongly dependent with no global predominant pattern [Fig. 8(d)]. The velocity components $u_z$ and $u_x$ at this point in state subspace display a spatially uncorrelated picture without predominant flow structure. The irregularly distributed flow is unstable in time and tries to reorganize into a structure of large-scale thermal circulation. The typical flow fields displayed in Fig. 8 indicate that pure one- and two-cell flow structures occur only at the fixed points of the drift vector field in state subspace (Fig. 6). Outside of these fixed points the flow is characterized by higher-order flow structures, a superposition of different flow patterns [Fig. 8(c)], and in particular by an increased plume activity. Hence, there is no clear one- or two-cell structure outside of the fixed points, without interaction of plumes with the bulk.

With respect to the phenomena of large-scale circulation, as discussed above, the heat transport is of special interest. The Nusselt number measures the ratio of actual heat flux due to advection $q_{\text{adv}}$ and conduction $q_{\text{cond}}$ to the heat flux of the purely conductive state $q_0$ and is given by

$$\text{Nu} = \frac{q_{\text{adv}} + q_{\text{cond}}}{q_0}.$$

A possible connection between global dynamics and the Nusselt number has been suggested previously by a number of authors [1,6,28]. Several experiments indicate that the Nusselt number is insensitive to changes in the large-scale dynamics. However, there are several numerical studies [29,30] and theoretical hints [31] indicating a sensitivity to changes assuming stress-free boundary conditions. Through our analysis we can combine generic flow structures with coordinates in state subspace. To provide information about the heat transport of dominant modes, we analyze the 2D temporal average of the Nusselt number conditionally averaged with respect to position in the state subspace of the amplitudes $\xi_{11}(t)$ and $\xi_{21}(t)$, given by $\langle \text{Nu}(t) \mid \xi_{11}(t), \xi_{21}(t) \rangle$. This quantity in combination with the flow examples of Fig. 8 reveals how effective the different flow structures are for cooling and heating. The temporal average of the Nusselt number conditionally averaged with respect to the one- and two-cell pattern (Fig. 9) increases for increasing amplitudes. This implies that for plume-dominated flow structures, where the instabilities interact frequently with the bulk, the Nusselt number is much higher. For the zero two-cell pattern domain there are two minima in the Nusselt number visible. For this domain in state subspace, the plumes do not reach far into the bulk, as seen in Fig. 8(a). The increased Nusselt number at high amplitudes, i.e., high magnitudes of the drift vector field, and the minima in the Nusselt number near the fixed points of the drift vector field point out the plumes that interact with the bulk increase the heat transport.

To extract further information about the Nusselt number with respect to the primary modes, we examine the PDF of the Nusselt number conditionally averaged with respect to the most probable configuration in state subspace. For this quantity we select a domain around the maxima of the PDF with respect to a threshold. The PDF of the Nusselt number conditionally averaged with respect to the isolated maximum of the clockwise and anticlockwise orientated one-cell circulation is displayed in Fig. 10. By varying the threshold, more or fewer fluctuations in the form of plumes influence the PDF. Instead of a Gaussian distribution, the
The stability and the statistical dependency with respect to different primary flow modes. We also connect these primary modes with the Nusselt number of the corresponding flow fields in order to analyze which structures are transporting the most amount of heat through the fluid layer. The comparison between the temporal evolution of the vertical velocity at the side walls at middepth and a reduced model of a finite set of modes shows that the global dynamics can be characterized by at least two modes. Analysis of a generic reversal also indicates that the amplitudes \( \xi_{11} \) and \( \xi_{21} \) dominate the temporal evolution of the LSC. Due to statistical analysis in state subspace spanned by two predominant modes, we are able to extract the dependency between both modes as well as the reversal path in state subspace. Special attention has been given to the characterization of typical reversal sequences. Our study indicates, unlike previous work [13–16], that a generic mechanism always involves transitions over higher-order modes. Such a reversal goes along with a breakdown of the LSC through a strong interaction of plumes with the bulk, an excitation of higher flow modes, a breakdown of this partly stable flow pattern, and a reestablishment of a reversed LSC. Thus the present paper allows us to characterize the dynamics and the irregular reorientation of the LSC. The interaction between the small-scale structures in form of plumes with the bulk leads to an increased heat transport; however, due to the rare occurrence of the plumes they do not change the contribution to the heat transport significantly. Furthermore, a major issue for future research would be to analyze a different parameter range of Rayleigh number and aspect ratio \( \Gamma \), to investigate which structures are dominating the global flow pattern. An important future challenge will be the connection between the primary modes and the governing equations in order to extract low-dimensional model equations to characterize the global dynamics of the flow field on the basis of the primary flow patterns.

**ACKNOWLEDGMENTS**

The work benefited from the constructive comments by the two anonymous referees. The authors also thank O. Kamps for helpful discussions within the framework of CeNoS.