

Wellen, Wogen, vogues, waves,
olas, ondas, undulations, ondata,
波 [bō], волна́, vaga, fala, κύμα

Phenomena and features

Christmas lecture U Thiele 2015



The Great Wave off Kanagawa, Hokusai, 1830–33,
Woodblock print,
wikipedia.org/wiki/The_Great_Wave_off_Kanagawa



Sound wave, Jean Shin,
<http://www.designsoak.com/modern-artist-jean-shin/>

Ethymology

- **wave (noun):** "moving billow of water," 1520s, alteration (by influence of wave (v.)) of Middle English *waw*, which is from Old English *wagian* "to move to and fro" (cognates: Old Saxon, Old High German *wag*, Old Frisian *weg*, Old Norse *vagr* "water in motion, wave, billow," Gothic *wegs* "tempest,"
- **wave (verb):** "move back and forth," Old English *wafian* "to wave, fluctuate" (related to *wæfre* "wavering, restless, unstable"), from Proto-Germanic *wab-* (cognates: Old Norse *vafra* "to hover about," Middle High German *waben* "to wave, undulate"), possibly from PIE root **webh-* "to move to and fro; to weave".
- **New Wave:** 1960, of cinema (from French *Nouvelle Vague*, late 1950s); 1976 as a name for the more restrained and melodic alternative to punk rock. [Cf. also **Neue Deutsche Welle**]

Source: <http://www.etymonline.com>, accessed 15.12.2015

Questions

- What kinds of waves do we know?
- Is there a difference between "proper waves" and "wave-like phenomena"?
- In which (scientific) fields do waves occur? "[Wave science](http://www.rochester.edu/newscenter/2014-lewis-henry-morgan-lecture-waves-an-anthropology-of-scientific-things) is the study of periodic, oscillating, and undulating phenomena—in different fields, from cosmology, to biology, oceanography, sport, and social science." (<http://www.rochester.edu/newscenter/2014-lewis-henry-morgan-lecture-waves-an-anthropology-of-scientific-things>, accessed 10.12.2015)
- How do we model waves, and classify them?

Wave phenomena

Vague de protestations, Hitzewelle, vague de froid, Monsterwelle, *vague de nostalgie*, *onde verte*, *зелёная волна́*, buckling, Haarwelle, la-ola-Welle, shock wave, seismische Welle, (Neue) Deutsche Welle, Rogue wave, kinematic waves, mechanical waves, electromagnetic waves, waves of excitement, chemische Wellen, Elliott wave (also cf. Alfvén, Bloch, de Broglie, Faraday, Gerstner, Kelvin, Lamb, Langmuir, Love, Mach, Rayleigh, Rossby, Stokes, Tollmien–Schlichting waves), Ortsgemeinde Wellen, Heilende Wellen, Lange Wellen (Konratieff-Zyklen), New Wave (also Cold, Dark, Doom, Electro, Ethereal, Gothic Wave), Lichtwellen, Radiowellen, Gravitationswellen, earthquake waves, matter waves, internal water waves, density waves, Schallwellen, surface waves, elastic waves, brain waves, Neutrinowellen, plasma waves, fashion waves, traffic flow waves, Kapillarwellen, ship waves, waves in solids (P-, S-waves)

(classical) History of Wave Science

- **Pythagoras** (c. 560-480 BC), studied relation of pitch and length of string in musical instruments
- **Giovanni Benedetti** (1530-90), **Isaac Beeckman** (1588-1637) and **Galileo** (1564-1642) established relationship between pitch and frequency
- **Joseph Sauveur** (1653-1716) showed that strings can vibrate simultaneously at a fundamental frequency and at integral multiples that he called harmonics -established **acoustics**
- **Isaac Newton** (1642-1727) calculated the speed of sound in his *Principia*
- **Daniel Bernoulli** (1700-82), **Leonard Euler** (1707-83) and **Jean d'Alembert** (1717-83) found first solution to linear wave equation
- **Joseph Fourier** (1768-1830) conjectured that arbitrary functions can be represented by superposition of infinite sum of sines and cosines (Fourier series) - proven by **Dirichlet**
- **John William Strutt (Lord Rayleigh)**, 1832-1901) treatise *Theory of Sound* (classical acoustics), **first theory related to nonlinear (soliton) waves 1879**
- Waves in Fluid Dynamics: Siméon-Denis **Poisson** (1781-1840), Claude Louis Marie Henri **Navier** (1785-1836), Augustin Louis **Cauchy** (1789-1857), Sir George Gabriel **Stokes** (1819-1903), Sir George Biddell **Airy** (1801-92)
- full unified theory of electromagnetism by **James Clerk Maxwell** (1831-79) in his *Treatise on Electricity and Magnetism*. All electromagnetic and optical phenomena including waves accounted for

source: http://www.scholarpedia.org/article/Nonlinear_wave, accessed 15.12.2015

Types of Waves

- ★ **Standing wave**
- ★ **Travelling wave**
- ★ **Modulated wave**
- ★ **Chaotic wave**
- ★ **Longitudinal/transversal/torsional Waves**
- ★ **Spiral wave, Scroll wave**
- ★ **Solitary wave (Soliton, dissipative soliton, Auto soliton)** - Humps (pulses), Kinks, Breathers (bions)
- ★ **Shock wave, Fronten**
- ★ **Linear/nonlinear waves**

Mechanical Waves

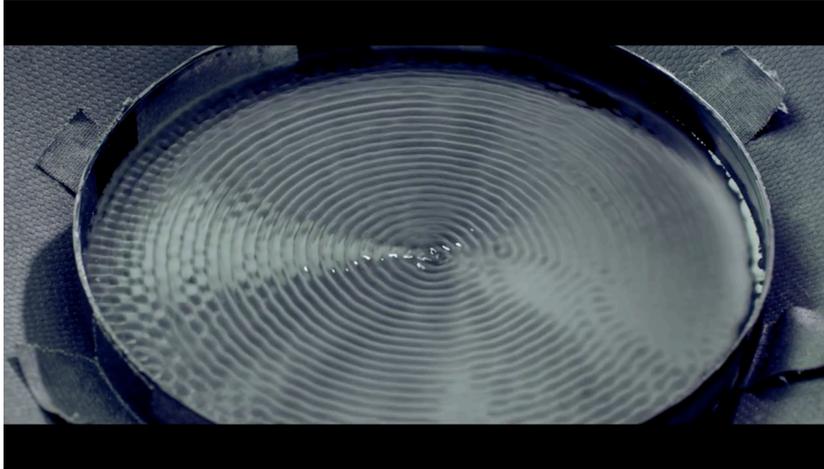
- wave on a rope (**Experiment**)
- density wave, acoustic wave (Flammrohr, **Exp**)
- Vibrating Plate (Chladnische Klangfiguren, **Exp**)
- Water waves
- elastic waves (e.g., seismic waves)

Vibrating Plate



source: <https://www.youtube.com/watch?v=wwJAgUBF4w>; attention ! Esoterics "The Sacred Geometry Of Sound", 3:38, last accessed 15.12. 2015

more Esoterics: Cymatics



source: CYMATICS: Science Vs. Music - Nigel Stanford, <https://www.youtube.com/watch?v=Q3oltPva9fs>, last accessed 18.12. 2015

Torsion-spring-coupled pendula I

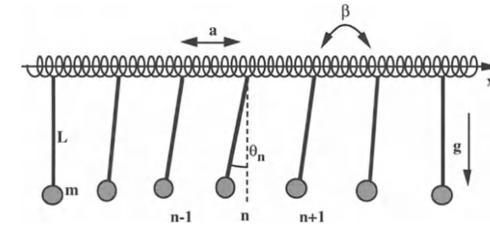


Fig.6.1. Sketch of the mechanical transmission line which consists of elastically coupled pendulums.

$$\frac{d^2\theta_n}{dt^2} + \omega_0^2 \sin\theta_n = \frac{c_0^2}{a^2} (\theta_{n+1} + \theta_{n-1} - 2\theta_n) \quad (n = 1, 2, \dots, N).$$

acceleration gravity torsion spring

Linearization: $\sin \theta \rightarrow \theta$

Harmonic time dependence: $\theta_n(t) \rightarrow q_n e^{i\omega t}$

Linear algebraic eigenvalue problem: $(\mathbf{A} - \lambda \mathbf{I}) \mathbf{q} = 0$ mit $\mathbf{q} = (q_1, q_2, q_3, \dots, q_N)$

➡ **Eigenmodes of mech. System** [Eigenvalues $\lambda = \omega^2$ Eigenvectors \mathbf{q}]

source partly: M Remoissenet, Waves called Solitons (Chap. 6), Springer (1999)

Eigenmodes of an mechanical system



(Counter)Example:
London Millenium
Bridge Opening

source: <https://www.youtube.com/watch?v=gQK21572oSU>, last accessed 21.12. 2015

Torsion-spring-coupled pendula II

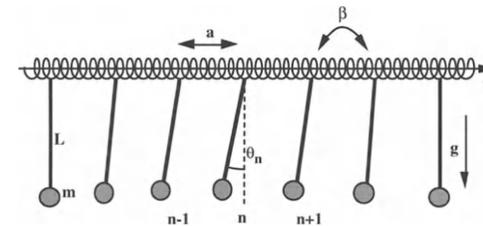


Fig.6.1. Sketch of the mechanical transmission line which consists of elastically coupled pendulums.

$$\frac{d^2\theta_n}{dt^2} + \omega_0^2 \sin\theta_n = \frac{c_0^2}{a^2} (\theta_{n+1} + \theta_{n-1} - 2\theta_n) \quad (n = 1, 2, \dots, N).$$

acceleration gravity torsion spring

Continuum limit: $\theta_n(t) \rightarrow \theta(x,t)$

$$\frac{\partial^2\theta}{\partial t^2} - c_0^2 \frac{\partial^2\theta}{\partial x^2} + \omega_0^2 \sin\theta = 0 \quad \text{a nonlinear wave equation: the sine-Gordon equation}$$

$$\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\phi}{\partial t^2} = \sin\phi$$

scaled

source: M Remoissenet, Waves called Solitons (Chap. 6), Springer (1999)

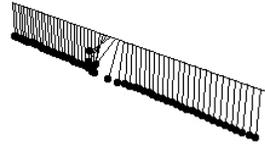
sine-Gordon equation - solitons

kink solution

$$\phi = 4 \arctan \exp \left\{ \pm (x - vt) / \sqrt{1 - v^2} \right\}$$

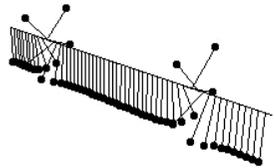
breather

$$\phi(x, t) = 4 \arctan \left[\left(\frac{\sqrt{1 - \omega^2}}{\omega} \right) \frac{\sin \omega t}{\cosh \sqrt{1 - \omega^2} x} \right]$$



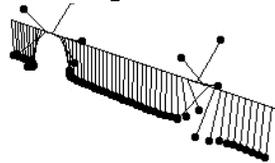
kink-kink collision

$$\phi(x, t) = 4 \arctan \left[\frac{v \sinh(x/\sqrt{1 - v^2})}{\cosh(vt/\sqrt{1 - v^2})} \right]$$

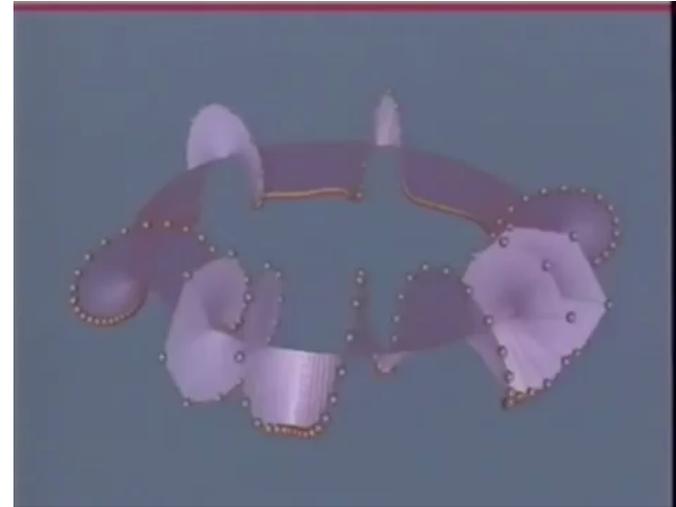


kink-antikink collision

$$\phi(x, t) = 4 \arctan \left[\frac{\sinh(vt/\sqrt{1 - v^2})}{v \cosh(x/\sqrt{1 - v^2})} \right]$$



sine-Gordon equation - more solitons



Source: visualization by Uwe Schwarz, <https://www.youtube.com/watch?v=Ozt1VkeK52E>, last accessed 20.12.2015

Shallow water waves



Stokes drift in shallow water waves,
with a wave length much longer than the water depth

Theory: The Korteweg-de Vries equation
derived from the full hydrodynamic equations
in a “shallow-water approximation”

Source: https://commons.wikimedia.org/wiki/File:Shallow_water_wave.gif

Korteweg-de Vries equation

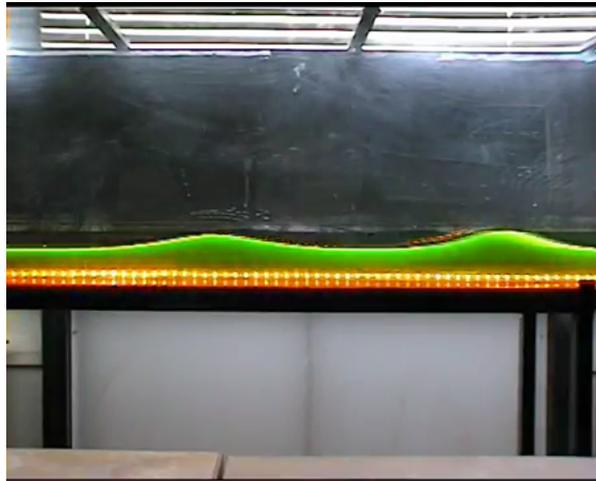
SixtySec Korteweg-de Vries & Solitons

Presenter: Prof. Boris A. Malomed



Source: Boris Malomed, Tel Aviv University, Israel, tutorials on Solitons (<http://www.explore-gate.com>),
hosted on <https://www.youtube.com/watch?v=WnJKl6lKwuQ>, last accessed 21.12.2015

Solitary waves - Solitons



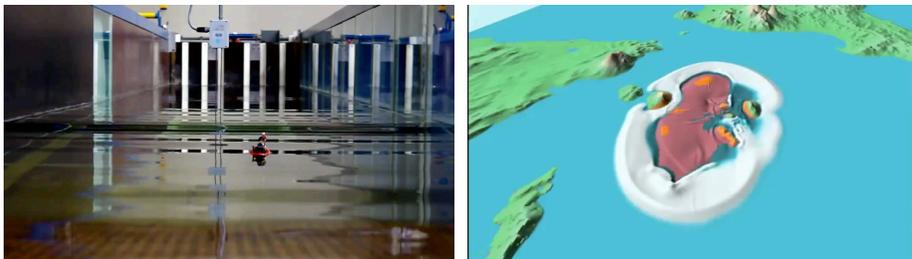
Source: Collision of KdV solitons, Laboratoire Interdisciplinaire CARNOT de Bourgogne, Équipe Solitons, Laser et Communications optiques, <https://www.youtube.com/watch?v=wEbYELtGZwI>, last accessed 20.12. 2015

(simple) Nonlinear wave equations

- **Hopf** equation (inviscid Burgers equation): $u_t + uu_x = 0$
- **Burgers** equation: $u_t + uu_x - au_{xx} = 0$
- **Fisher** equation: $u_t - u_{xx} - u(1-u) = 0$
- **Sine Gordon** equation: $u_{tt} = au_{xx} + b\sin(\lambda u)$
- **Cubic Schrödinger** equation: $i u_{tt} + u_{xx} + q|u|^2 u = 0$
- **Korteweg-de Vries** $u_t + uu_x + bu_{xxx} = 0$
- **Boussinesq** equation: $u_{tt} - u_{xx} + 3uu_{xx} + au_{xxxx} = 0$
- Nonlinear wave equation of general form: $u_{tt} = [f(u)u_x]_x$
- **Thin film equation**: $u_t + [u^3(u_{xxx} - u_x) + bu^3]_x = 0$
- Coupled systems, reaction-diffusion systems, higher derivatives, integro-differential equations, ...

source: in part from http://www.scholarpedia.org/article/Nonlinear_wave

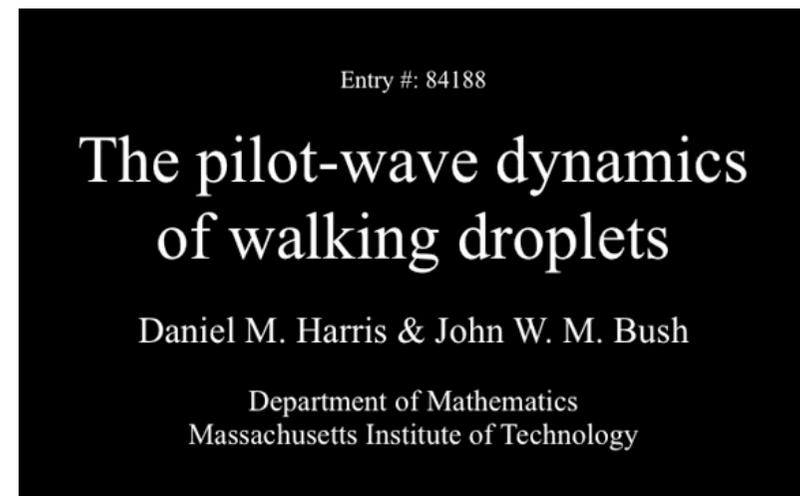
Monster waves and Tsunamis



Source: (left) <https://www.youtube.com/watch?v=u0psJp6I2Z0>; (right) This movie shows a physics-based computer simulation of the 1883 Krakatoa eruption, lateral blast, pyroclastic flow and tsunamis. <https://www.youtube.com/watch?v=y2lxUyF7ip4>, last accessed 17.12. 2015

Bouncing, walking droplets

Particle-wave dualism in hydrodynamics?



source: first Exp. by Yves Couder's group, <https://www.youtube.com/watch?v=nmC0ygr08tE>, last accessed 15.12. 2015

Collective behaviour in non-mechanical systems

Many individual agents interact with neighbours following local rules

E.g. chemical reactants, cars, biological cells, molecular motors, ants, sand, people, ...

Crowd dynamics - "la ola"



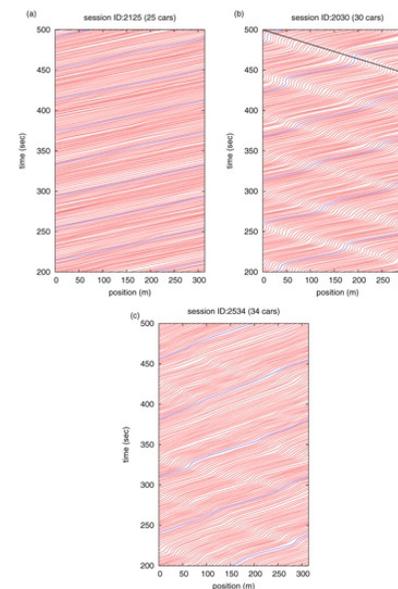
Source: (left) De Kreuners 15/12/2012 - mexican wave by JoRodder, <https://www.youtube.com/watch?v=rmpBM93Shlw> (right) <http://www.sueddeutsche.de/wissen/verhaltensbiologie-la-ola-bei-praeriehunden-1.1857632>, last accessed 16.12. 2015

Spontaneous Traffic jams



Source: (left) Experimental evidence for the physical mechanism of forming a jam. "The Mathematical Society of Traffic Flow", Yuki Sugiyama et al., New Journal of Physics, 2008, Multimedia supplement., <https://www.youtube.com/watch?v=Suugn-p5C1M>; (right) Traffic Waves, Robin Guest <https://www.youtube.com/watch?v=19S3OdK6710>, last accessed 16.12. 2015

Space-time plots



Space-time diagrams for three sessions (2125, 2030 and 2534) with 25, 30 and 34 cars. The vertical axis is time (s) and the horizontal axis is position (m).

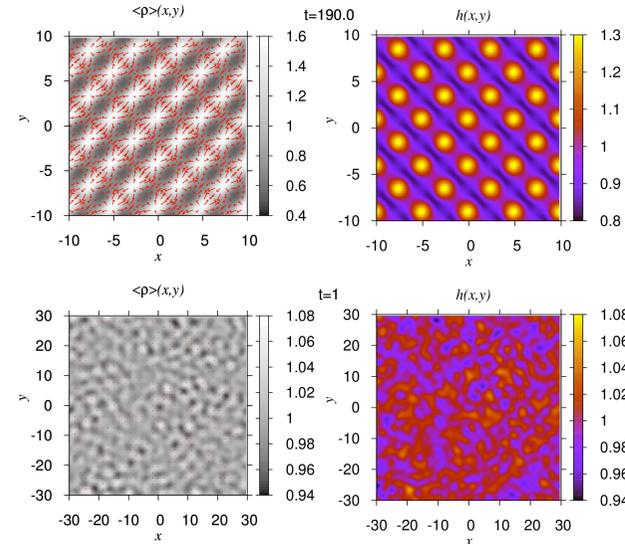
Source: Phase transition in traffic jam experiment on a circuit, Shin-ichi Tadaki, Macoto Kikuchi, Minoru Fukui, Akihiro Nakayama, Katsuhiro Nishinari, Akihiro Shibata, Yuki Sugiyama, Taturu Yosida and Satoshi Yukawa; *New Journal of Physics*, Volume **15**, 2013

Chemical waves



Source: (left): **Belousov-Zhabotinsky Reaction**: nater06: "I performed this reaction as an assignment after it was referenced in Ilya Prigogine's book "The End of Certainty" as an example of a chemical reaction that gained new properties when far from equilibrium; <https://www.youtube.com/watch?v=1Ba4kgXl4Cg>; (right): Cornus Ammonis, variant of Tim Hutton's Mutually-Catalytic Spots system (two coupled Gray-Scott systems [cf. <http://mrob.com/pub/comp/xmorphia/index.html>]); <https://www.youtube.com/watch?v=VPO9wDJ28pc>, last accessed 18.12. 2015

Carpet of micro-swimmers on liquid film



standing waves

chaotic waves

Source: Model for bacterial carpets, A Pototsky, U. Thiele, H. Stark (2015, unpublished)

Bei Interesse an mehr Nichtlinearem siehe:

Bücher von **Prigogine, Bestehorn, Haken, Nicolis, Strogatz, Argyris, Ebeling**

Nichtlineares Praktikum im Bachelor- oder Masterstudium
 Bachelorseminar (WS 2016/17)
 Bachelorarbeiten (Theorie ITP, Experiment AP)
 Masterspezialisierung: Nichtlineare Physik
 Masterseminar "Nichtlineare Physik"



Ich wünsche Ihnen geruhsame
 Physikalische Feiertage

es folgt der Abspann: Die "Skalierungswelle"

"Scaling waves" or the psychedelic xmas-card generator



$$Z_{n+1} = Z_n^2 + C$$

$$Z_0 = 0$$

black: absolute value of Z_n remains bounded

source: Deepest Mandelbrot Set Zoom Animation ever - a New Record! 10^{275} ($2.1E275$ or 2^{915}) Orson Wang; <https://www.youtube.com/watch?v=0jGao87u3A>, last accessed 19.12. 2015 ; or <http://fractaljourney.blogspot.de>