Investigation of Theories beyond the Standard Model

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The fundamental constituents of matter and the forces between them are splendidly described by the Standard Model of elementary particle physics. Despite its great success, it will be superseded by more comprehensive theories that are able to include phenomena, which are not covered by the Standard Model. Among the attempts in this direction are supersymmetry and Technicolor models. We report about our non-perturbative investigations of the characteristic properties of such models by numerical simulations on high-performance computers.

1 Introduction

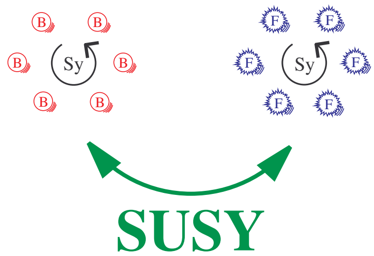
The physics of elementary particles has a marvellous theoretical framework available, which is called the *Standard Model*. It accurately describes a vast number of phenomena and experiments. In the Standard Model the fundamental interactions among particles are described in terms of gauge field theories, which are characterized by an infinite-dimensional group of symmetries.

Despite its great success it is evident that the Standard Model does not offer a complete description of the fundamental constituents of matter and their interactions. To mention just some of the arguments for this: (1) astrophysical observations have revealed that the universe contains a huge amount of dark matter, that is not described by the Standard Model, (2) it has been discovered experimentally that neutrinos have a finite mass, whereas the Standard Model requires them to be massless, (3) there is a hierarchy of mass scales between electroweak phenomena (e. g. W- and Z-masses) and light constituents (e. g. quark and lepton masses), which the Standard Model cannot explain in a natural way. A central question of present-day's elementary particle theory thus concerns the physics beyond the Standard Model. Some attempts in this direction are Grand Unified Theories, supersymmetric models, Technicolor, Supergravity and Superstring theories. In this article we will discuss work of our collaboration concerning supersymmetry and Technicolor Models.

2 Supersymmetric Yang-Mills Theory

The fundamental constituents of matter, quarks and leptons, are fermions, characterized by half-integer spin, and obeying the Pauli principle. The forces among them are mediated by bosons, which carry integer spin, like the photons, W- and Z-bosons for the electroweak interactions and the gluons for the strong interactions. Various symmetries are known that play important roles for physics on the fundamental level, and that are related to conservation laws via Noether's theorem. These symmetries, which can be described mathematically in terms of groups, relate fermions to fermions and bosons to bosons.

Supersymmetry goes beyond the concept of ordinary symmetries. It relates bosons with fermions, and in its mathematical description requires concepts exceeding group theory.



Particle multiplets of supersymmetry contain members with different spins, in particular both bosons and fermions. In the case of unbroken supersymmetry the particles in a supermultiplet would all have the same mass.

Supersymmetric extensions of the Standard Model, containing in addition to the particles of the Standard Model their superpartners, are candidates for models that describe the unification of forces at very high energies and supply dark matter particles. Since supermultiplets with degenerate masses have not been observed in nature, supersymmetry must be broken by some mechanism in these models.

Many of the properties of supersymmetric field theories have been investigated by means of perturbation theory or semiclassical methods. There are, however, important non-perturbative properties that are not accessible to these methods, in particular the masses of bound states and the nature of phases of the theories. The method of choice for studying these central characteristics is the numerical simulation on high performance computers. To this end space-time is discretized on a four-dimensional lattice, where the variables of the field theory are defined.

The object of our investigations is the $\mathcal{N} = 1$ supersymmetric Yang-Mills (SYM) theory. It represents the simplest field theory with supersymmetry and local gauge invariance, and it is contained in every supersymmetric extension of the Standard Model as a sub-sector. SYM theory is the supersymmetric extension of Yang-Mills theory with gauge group SU(N_c). It describes the carriers of gauge interactions, the "gluons", together with their superpartners, the "gluinos", forming a massless vector supermultiplet. The gluons are represented by the non-Abelian gauge field $A_\mu^a(x)$, $a = 1, \ldots, N_c^2 - 1$. The gluinos are massless Majorana fermions, described by the gluino field $\lambda^a(x)$ obeying $\bar{\lambda} = \lambda^T C$ with the charge conjugation matrix C, thus being their own antiparticles. Gluinos transform under the adjoint representation of the gauge group, so that the gauge covariant derivative is given by $\mathcal{D}_\mu \lambda^a = \partial_\mu \lambda^a + g\, f_{abc} A_\mu^b \lambda^c$.

In the continuum the (on-shell) Lagrangian of the theory is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{\mathrm{i}}{2} \bar{\lambda}^a \gamma_\mu (\mathcal{D}_\mu \lambda)^a \,, \tag{1}$$

where $F^a_{\mu\nu}$ is the non-Abelian field strength. Adding a gluino mass term $m_{\tilde{g}}\ \bar{\lambda}^a\lambda^a$, which is necessary in view of the numerical simulations, breaks supersymmetry softly.

SYM theory is similar to Quantum Chromodynamics (QCD), the theory of the strong interactions of nuclear matter. The essential differences are that the gluinos are Majorana fermions and that they are in the adjoint representation of the gauge group, in contrast to the quarks of QCD.

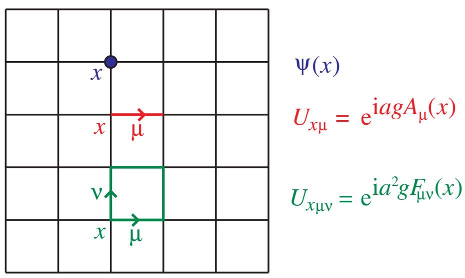
Non-perturbative problems concerning fundamental properties of SYM theory are (1) Is chiral symmetry broken spontaneously as predicted, associated with a gluino condensate $\langle\lambda^a\lambda^a\rangle \neq 0$? (2) Are static quarks confined? (3) Is supersymmetry broken spontaneously? (4) Does a supersymmetric continuum limit exist on the lattice? (5) Do the bound states form supermultiplets? (6) Are the predictions from effective Lagrangeans correct?

We have investigated SYM theory with gauge group SU(2) in recent years and are presently focussing on SU(3). Central aspects of our studies include the spectrum of lightest particles, supersymmetric Ward identities and the phases of the model. For our recent publications see Refs. 1–6.

2.1 Supersymmetry on the Lattice

In order to study a field theory by numerical simulations, the underlying space-time continuum has to be approximated by a lattice. The lattice spacing a provides a momentum cut-off, so that possible infinities, coming from the large-momentum region, are regulated. Expectation values of observables are calculated in terms of functional integrals à la Feynman. Their numerical evaluation with Monte Carlo methods requires that the integrands are real. This is achieved by an analytical continuation to imaginary times, $t = -\mathrm{i}\tau$, leading to *Euclidean lattice field theory*.

On the lattice the gauge field is given by link variables $U_{x\mu} \in \mathrm{SU}(N_c)$, and the gauge field strength is represented by the product $U_{x\mu\nu} \equiv U_p$ of the gauge link fields along a plaquette $p = (x, \mu\nu)$. Fermion fields $\psi(x)$ are defined on lattice points.



Supersymmetry is generically broken by the discretization of space-time on a lattice[7]. Therefore a question of conceptual relevance is whether supersymmetry can be restored in the continuum limit ($a \to 0$). In our numerical calculations we use a lattice action proposed by Curci and Veneziano[8], which is built in analogy to the Wilson action of QCD for the gauge field and Wilson fermion action for the gluino. Both supersymmetry and chiral symmetry are broken on the lattice, but they are expected to be restored in the continuum limit if the gluino mass is tuned to zero.

The Curci-Veneziano action for SYM theory on the lattice is given by $S = S_g + S_f$, where

$$S_g = -\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\,U_p \tag{2}$$

is the gauge field action with inverse gauge coupling $\beta = 2N_c/g^2$, and

$$S_f = \frac{1}{2}\sum_x \left\{ \bar{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\bar{\lambda}^a_{x+\hat{\mu}} V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x + \bar{\lambda}^a_x V^T_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}} \right] \right\} \tag{3}$$

is the fermion action, where $V_{ab,x\mu} = 2\,\mathrm{Tr}\,(U^\dagger_{x\mu} T_a U_{x\mu} T_b)$ is the gauge field variable in the adjoint representation (T^a are the generators of SU(N_c)), and the hopping parameter κ is related to the bare gluino mass via $\kappa = 1/(2m_0+8)$. When the fermion action is written in the form $S_f = \frac{1}{2}\bar{\lambda}Q\lambda = \frac{1}{2}\lambda^T M\lambda$ with $M = CQ$, the functional integral $Z = \int [DU][D\lambda]\,\mathrm{e}^{-(S_g+S_f)}$ involves the fermionic part

$$\int [D\lambda]\,\mathrm{e}^{-S_f} = \mathrm{Pf}(M) = \pm(\det Q)^{1/2}\,, \tag{4}$$

resulting in a *Pfaffian*. For finite lattice spacings a the Pfaffian is not always positive and its sign has to be taken into account separately. By monitoring the sign of the Pfaffian we check that practically no sign problem occurs because the positive contributions dominate.

The exponent 1/2 of $\det Q$ can be interpreted as corresponding to a flavour number $N_f = 1/2$ of Dirac fermions. The gauge configurations for this fractional flavour number can be created, for instance, by the *two-step polynomial Hybrid Monte Carlo (TSPHMC)*[9] or the *rational Hybrid Monte Carlo (RHMC)* algorithm, both of which we use for Monte Carlo updating.

2.2 Light Particle Spectrum in SYM Theory

Similar to QCD, SYM theory is asymptotically free at high energies and strongly coupled at low energies. It is expected that due to confinement the particles of the model are colour-neutral bound states of gluons and gluinos. Because of supersymmetry they should belong to mass degenerate supermultiplets. It is a central aim of our studies to investigate whether this expectation holds.

Based on effective Lagrangeans, predictions about the composition of the lightest supermultiplets have been made[10,11]. A first chiral supermultiplet should consist of a scalar meson a–f_0, a pseudoscalar meson a–η', and a gluino-glue bound state $\tilde{g}g$, being a spin 1/2 Majorana particle. The corresponding interpolating fields are $\bar{\lambda}^a\lambda^a$, $\bar{\lambda}^a\gamma_5\lambda^a$ and $\sigma_{\mu\nu}F^a_{\mu\nu}\lambda^a$. The prefix a of the meson-names indicates that their constituents are in the adjoint representation. An additional chiral supermultiplet is predicted to consist of a scalar (0^+) glueball, a pseudoscalar (0^-) glueball, and another gluino-glue bound state $\tilde{g}g$. Fig. 1 illustrates the elementary constituents at high energies and their bound states in the low energy regime.

The masses of the bound states are obtained from an analysis of the corresponding correlation functions. We also employed more elaborate variational methods. A circumstance that makes the calculations considerably more demanding than in QCD is the fact that the mesons are flavour diagonal, and consequently their correlation functions always contain disconnected fermionic contributions, whose numerical evaluation is rather laborious.

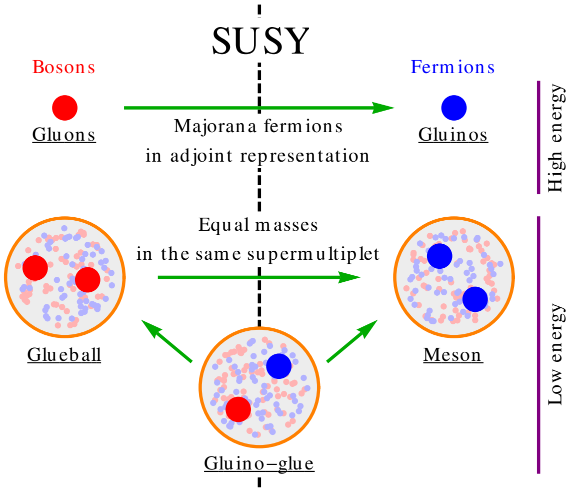


Figure 1. Constituents and bound states in SYM

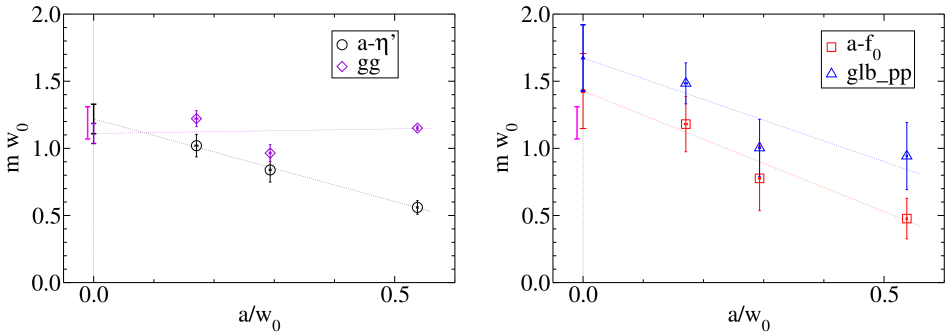


Figure 2. Light particle masses in SU(2) SYM theory a function of the lattice spacing a and extrapolated to the continuum. Masses and lattice spacings are given in units of the scale parameter w_0 obtained from the Wilson flow.

In the case of gauge group SU(2) we have obtained results for low-lying masses on lattices of size $16^3 \times 32$, $24^3 \times 48$ and $32^3 \times 64$ at three values of the lattice spacing, namely $a = 0.087$, 0.054 and 0.036 fm in units of QCD.

Two kinds of extrapolations are performed. First, at fixed lattice spacing a, determined by the gauge coupling β, the masses are extrapolated to the limit of vanishing renormalized gluino mass $m_{\tilde{g}} = 0$. This limit corresponds to a particular value of the hopping parameter $\kappa_c(\beta)$, which can be determined either with the help of the adjoint pion mass $m_{\text{a-}\pi}$, making use of the relation[12] $m^2_{\text{a-}\pi} \propto m_{\tilde{g}}$, or by means of supersymmetric Ward identities. Both methods appear to be consistent with each other up to lattice artefacts.

The second extrapolation is the one towards the continuum limit. Fig. 2 shows the masses at three values of the lattice spacing a and their extrapolations to the continuum

$a = 0$. The notoriously difficult glueball has the noisiest signal and largest errors, which might be underestimated. With this reservation the results are consistent with the formation of a degenerate supermultiplet.

Currently we are investigating the next higher masses, which should belong to the members of a second supermultiplet, and we have obtained preliminary results. The focus of our present calculations is on SYM theory with gauge group SU(3), which contains the gluons of QCD and their fermionic superpartners. Compared to the case of SU(2), the SU(3) SYM theory shows new physical aspects, e. g. new types of bound states and CP-violating phases.

3 Technicolor Candidates

The Higgs particle, which has been found in 2012 at the LHC, plays an essential role in the Standard Model. The associated Higgs field is responsible for the masses of quarks, leptons, and the W- and Z-bosons, which mediate the weak interactions. The Higgs boson has a strange singular position in the Standard Model: it doesn't fit in the matter particles, which are all fermionic, neither does it fit in the other bosons, which are gauge particles. Moreover, in the Standard Model the Higgs mass is not protected against large radiative corrections, and its relative small value cannot be explained in a natural way. This gave rise to the idea that the Higgs particle might be a bound state of fermions, which interact via new strong interactions on a scale of order 200 GeV. *Technicolor models* are attempts to put this scenario into effect. Classes of models that found particular interest due to phenomenological constraints are theories with an infrared fixed point or with a walking coupling. Theories with an infrared fixed point show scale-invariant (conformal) behaviour at large distances, while walking theories have a nearby infrared fixed point. In both cases the coupling strength varies only slowly over a large range of scales, in contrast to the running coupling of QCD.

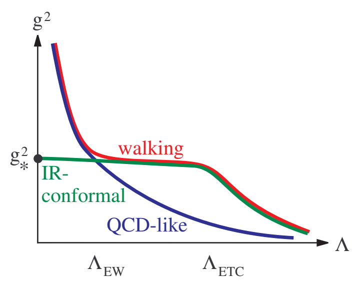


Figure 3. A sketch of the behaviour of the coupling strength g^2 as a function of the energy scale Λ for QCD-like, walking, and infrared conformal theories.

For gauge theories coupled to fermions it is of fundamental importance to know to which scenario they belong. This strongly depends on the number N_f of fermion flavours. In general, theories with small N_f behave QCD-like, and above a certain value of N_f infrared conformal behaviour sets in. Whereas for fermions in the fundamental representation of the gauge group the border is uncomfortable high ($N_f \approx 10$), it has been predicted to be significantly smaller for fermions in the adjoint representation.

This appealing feature has motivated our collaboration to investigate SU(2) gauge theories with fermions in the adjoint representation in view of their scaling behaviour. We have studied $N_f = 2$, 3/2, 1 and 1/2, where half-integer flavours mean Majorana fermions. A characteristic feature is the dependence of bound state masses on the fermion mass m_r. In infrared conformal theories, these masses commonly scale to zero with m_r according to $M \propto m_r^\alpha$ with some exponent α.

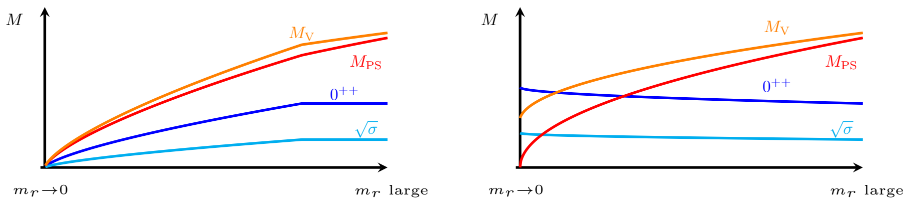


Figure 4. Typical behaviour of bound states masses as a function of the fermion mass m_r in infrared conformal theories (left) and QCD-like theories (right).

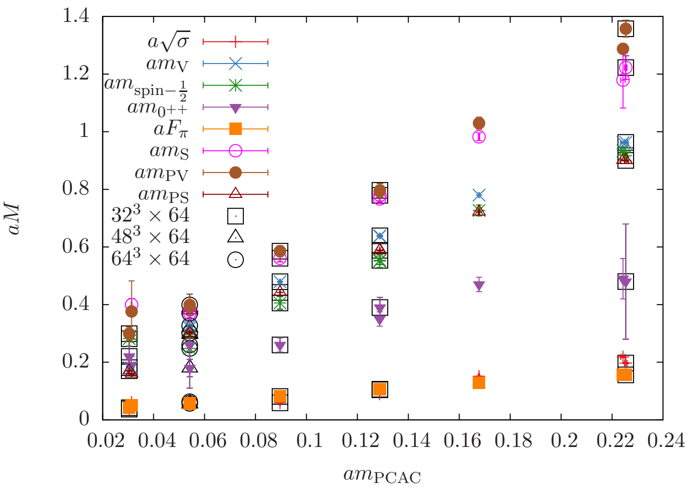


Figure 5. Masses of various bound states as a function of the fermion mass $m_r \equiv m_{PCAC}$ in SU(2) gauge theory with $N_f = 2$ flavours of fermions in the adjoint representation.

The case of $N_f = 1/2$ is SYM theory, which behaves QCD-like. On the other hand, for $N_f = 2$ fermion flavours the scaling of bound states masses (see Fig. 5), and other observables indicate, that this theory belongs to the infrared conformal scenario[13,14].

For the theories with $N_f = 1$ and $3/2$ we found indications that they are infrared conformal, too[15,16]. Our preliminary results are currently explored more closely.

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