Supporting Information for: Instabilities of layers of deposited molecules on chemically stripe patterned substrates: Ridges vs. drops

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1 Implementation in AUTO-07p

For the treatment with the continuation toolbox AUTO-07p, we transform the set of equations that determines the steady state solutions *and* the accompanying linear stability problem into a set of first order ordinary differential equations. The application of the continuation approach to a similar thin film equation is outlined in Ref. 1 while explanations of technical details and example codes can be found in the tutorials `drop`, `hetdrop`, and `lindrop` of Ref. 2.

In particular, we transform the nonautonomous Eq. (5) of the main text

$$\partial_x^2 h_0(x) + \mathit{\Pi}(h_0, x) + C = 0$$

that is of second order and the nonautonomous Eqs. (7)

$$\begin{aligned}\beta h_1 = &- Q(h_0)(\partial_x^2 - q^2)\\ &\times \left[(\partial_x^2 - q^2)h_1 + (\partial_h \mathit{\Pi}(h_0, x))h_1\right]\\ &- (\partial_x Q(h_0))\partial_x \left[(\partial_x^2 - q^2)h_1 + (\partial_h \mathit{\Pi}(h_0, x))h_1\right]\end{aligned}$$

that is of fourth order into an autonomous system of seven first order ODEs on the interval $[0, 1]$. To this end, we first define the independent variable $\xi := x/L$ with L denoting the physical domain size. Next, we introduce the variables

$$
\begin{aligned}
u_1(\xi) &= h_0(L\xi) - \bar{h} \ , && (1)\\
u_2(\xi) &= \left.\frac{\mathrm{d}h_0}{\mathrm{d}x}\right|_{x=L\xi} \ , && (2)\\
u_3(\xi) &= h_1(L\xi) \ , && (3)\\
u_4(\xi) &= \left.\frac{\mathrm{d}h_1}{\mathrm{d}x}\right|_{x=L\xi} \ , && (4)\\
u_5(\xi) &= \left.\frac{\mathrm{d}^2h_1}{\mathrm{d}x^2}\right|_{x=L\xi} \ , && (5)\\
u_6(\xi) &= \left.\frac{\mathrm{d}^3h_1}{\mathrm{d}x^3}\right|_{x=L\xi} \ , && (6)\\
u_7(\xi) &= L\xi \ . && (7)
\end{aligned}
$$

Here, $\bar{h}$ denotes the mean film thickness. With the notation $\dot{u}_i(\xi) = \mathrm{d}u_i(\xi)/\mathrm{d}\xi$, the system of first order ODEs reads

$$
\begin{aligned}
\dot{u}_1 =& Lu_2 && (8)\\
\dot{u}_2 =& -L[\Pi(\bar{h}+u_1, u_7) + C] && (9)\\
\dot{u}_3 =& Lu_4 && (10)\\
\dot{u}_4 =& Lu_5 && (11)\\
\dot{u}_5 =& Lu_5 && (12)\\
\dot{u}_6 =& L\Big\{ -\frac{\beta u_3}{Q_0} + q^2 u_5 - \partial_x^2(h_1 \partial_h \Pi_0)\\
& - \frac{\partial_x Q_0}{Q_0} \left[(u_6 - q^2 u_4 + \partial_x(h_1 \partial_h \Pi_0)\right]\\
& + q^2 \left[u_5 - q^2 u_3 + \Pi'(\bar{h}+u_1)u_3\right] \Big\} && (13)
\end{aligned}
$$

$$
\dot{u}_7 = L \quad (14)
$$

with $Q_0 = Q(u_1 + \bar{h})$ and

$$\partial_x(h_1\partial_h \Pi_0) = \Pi''(\bar{h} + u_1, u_7)u_2u_3 + \Pi'(\bar{h} + u_1, u_7)u_4 + \Pi'_x(\bar{h} + u_1, u_7)u_3 \ , \tag{15}$$

$$\begin{aligned}\partial_x^2(h_1\partial_h \Pi_0) = {} & \Pi'''(\bar{h} + u_1, u_7)u_2^2u_3 \\ & + \Pi''(\bar{h} + u_1, u_7)(\partial_x^2 h_0)u_3 \\ & + 2\Pi''(\bar{h} + u_1, u_7)u_2u_4 \\ & + \Pi'(\bar{h} + u_1, u_7)u_5 \\ & + 2\Pi'_x(\bar{h} + u_1, u_7)u_4 \\ & + 2\Pi''_x(\bar{h} + u_1, u_7)u_2u_3 \\ & + \Pi'_{xx}(\bar{h} + u_1, u_7)u_3 \ .\end{aligned} \tag{16}$$

In the last two equations, primes denote derivatives w.r.t. h while the index x denotes a derivative w.r.t. x at constant h. The equation for $x = u_7$ allows to write the heterogeneity function in an autonomous way. This is necessary because AUTO-07p only solves autonomous ODEs.

References

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(2) Thiele, U., Kamps, O., Gurevich, S. V., Eds. *Münsteranian Torturials on Nonlinear Science: Continuation*; CeNoS: Münster, 2014; `http://www.uni-muenster.de/CeNoS/Lehre/Tutorials`.